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TRANSLATION

PRINCIPLES OF RADIO DIRECTION FINDING
(CHAPTERS 9, 10, and 11)

By

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FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE
OHIO

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EDITED MACHINE TRANSLATION

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By: I. S. Kukes and M. Ye. Starik

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

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А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
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К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH

DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

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rot	curl
lg	log

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Principles of Radio Direction Finding,
"Soviet Radio" Publishing House,
Moscow, 1964.
Pages: 517-541 and 636-640.

CHAPTER 9

TESTS OF RADIO DIRECTION FINDERS

List of Designations Appearing in Cyrillic

$k_{гн}$ = k_{sf} = scaling factor

в = fair = fairlead
вых = out = output

и = i = (definition undetermined)

к = comp = compensating

макс = max = maximum

н = load
нс = asym = asymmetric

п = f = field coil

Р, р = loop

Ф = fd = feeder

э = st = standard

В = V = variometer

ГСС = SSG = standard signal generator

З = ground

Preliminary tests of radio direction finders are performed in laboratories, final ones are performed in real operating conditions of the direction finder.

9.1. Laboratory Tests of Direction Finders with a Rotating Loop

Separate parts of the direction finder (the loop, variometers, etc.) require no special tests other than normal ones — measurement of inductance, capacitance, resistance, and coupling coefficient. We shall not dwell here on methods of measurement of these magnitudes.

During laboratory testing of the direction finder as a whole by a generator of standard signals there is required, analogously to normal measurement of receivers, use of an equivalent antenna. A peculiarity of the given case is that receiver-direction finder is fed simultaneously from two antennas: a loop and an open antenna, where the virtual height of the loop changes in a wide range with change of wavelength, and the phase of the emf induced in it differs by 90° from the phase of the emf in the antenna. Furthermore, ordinary generators of standard signals have an asymmetric output (one pole usually is grounded). Connection of output terminals of the generator to the loop creates a symmetry of its circuit, which may not correspond to real operating conditions of the loop.

In Fig. 9.1 there is presented the circuit of the equivalent of the antenna and the loop, considering these peculiarities. Parameters of the circuits are selected in such a manner that $L'_2 + L''_2 \ll L_0$, where L_0 – inductance of the loop; L_a, C_a, R_a, C_{fair} – inductance, capacitance, and resistance of the antenna and capacitance of its fairlead. Under these conditions the receiver has normal load both from the loop and from the antenna.

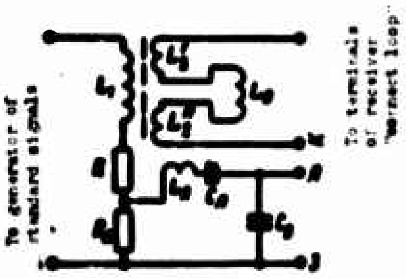

Fig. 9.1. Diagram of equivalent of antenna and loop.

Then we select $R \gg \omega L_1$; then the current through winding L_1 with sufficient accuracy (with error of 1%, if $R > 7\omega L_1$) can be expressed

$$\dot{I}_1 = \frac{E}{R},$$

where E – output voltage of generator.

The emf induced in coils L'_2 and L''_2, will be

$$E'_p = j\omega M I_1 = j\frac{\omega M}{R}E,$$

and voltage on resistance R_a, corresponding to the emf in the antenna, is equal to

$$E'_a = \frac{R_a}{R}E.$$

Coupling between coils L_1 and L'_2-L''_2 is made variable by sine law

$$M = M_{макс} \sin\theta.$$

During real work the emf in the frame is

$$E_p = jEh_p \sin\theta,$$

the emf in the antenna is

$$E_a = Eh_a.$$

We equate $E'_a = E_a$, $E'_{loop} = E_{loop}$ and $\alpha E = E$, where α – factor, which is conveniently selected equal to any round number (1, 2, 5, ...1/2, 1/5, 1/10, etc.). From this we find

$$\alpha = \frac{\omega M_{max}}{h_p R} = \frac{R_0}{h_a R} \text{ and } \frac{\omega M_{max}}{R_0} = \frac{h_p}{h_a}.$$

Since h_{loop} is proportional to frequency $h_{loop} = \frac{2\pi SN}{\lambda} = \frac{\omega SN}{3 \cdot 10^8}$, the lase equality is realizable in the whole range of frequencies. From it we find M_{max}, after which, given α, we find R. The reading on the divider dial of the generator of standard signals, multiplied by α, gives field strength in microvolts/meter.

By this circuit we can perform the following tests:

1. Determining sensitivity of the radio direction finder, i.e., the field strength which is required to ensure possibility of direction finding with error not exceeding a given value. For this, there is determined that voltage from the generator of standard signals at which bearing is read with the given accuracy. From the voltage field strength is calculated.

2. Check of exactness of determination of direction. Switching on the direction finder, we find field strength E_1 and E_2 in two positions, corresponding to determination of the direction, with constant output voltage. Depending on the scheme for determining direction these positions can be established either in the receiver itself by turning the variometer, switch, and so forth, or by turning the loop. In the last case in the test circuit turn of the loop is replaced by turn of variometer L'_2-L''_2 from the position corresponding to $+M_{max}$ to position $-M_{max}$. Relation $\frac{E_1}{E_2}$ characterizes exactness of determination of direction.

3. Check of compensation for antenna effects. The problem is to determine the relative emf of the antenna effect which can be compensated. It, obviously, is equal to the maximum emf created by the compensator. To determine this value we determine field strength E_0, creating normal output voltage with the position of the compensator, corresponding to zero emf of compensation. Then we turn variometer L'_2-L''_2 until we obtain zero emf in the loop circuit and the place compensator in the position, giving maximum compensation emf. In this position we again determine field strength E_{comp}, giving the same output voltage. Ratio $\frac{E_0}{E_{comp}}$ gives the value we sought.

4. It is possible to check remaining characteristics of the receiver (selectivity, fidelity, and so forth).

Testing by the two-signal method is accomplished with two equivalents, whose inputs are connected to two generators, and outputs are parallel-connected. Resistances and reactances of the equivalent should be doubled.

Another method of laboratory testing consists of placing the loop in a magnetic field, which is created by current in a horizontal rectilinear wire (line) (Fig. 9.2).

This test should be conducted in a shielded chamber, since during tests with the loop connected (and not with its equivalent, as in the preceding method) external interferences hamper tests a great deal. At a certain distance d from the chamber ceili: ; we stretch a rectilinear wire, which at one end is joined by a shielded cable to the generator of standard signals, and on the other through resistance R to the metal wall of the chamber.

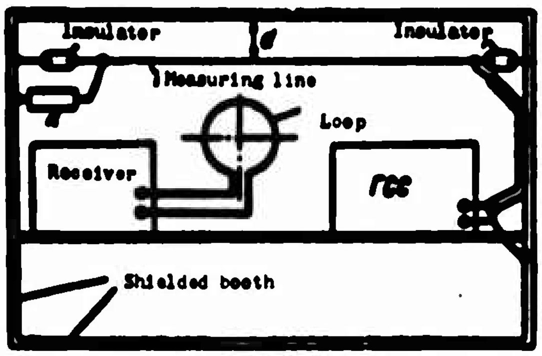

Fig. 9.2. Measuring line for testing a direction finder.

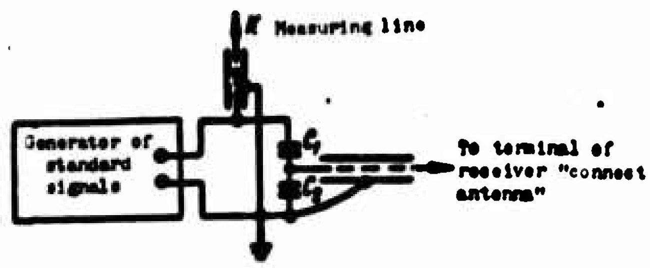

Fig. 9.3. Voltage divider.

The purpose of resistance R is to provide in the wire a traveling wave of current. In traveling wave conditions current in wire, and, consequently, magnetic field strength around it depends little on frequency. Magnetic field strength in these conditions also does not depend strongly on shift of the observation point along the wire.

Under wire there is placed the loop direction finder being tested. With rotation of the loop the minimum emf is induced in it at the time when its plane is perpendicular to the wire.

Magnetic and electrostatic fields of a rectilinear wire at a small distance from this wire do not have as simple a relationship to one another as in the zone of radiation. Therefore, use of the open antenna of a direction finder in its normal position can lead to a relationship of emf's induced in the antenna and loop, absolutely different from the relationship in real conditions. For testing it is necessary to use as the antenna a special section of rectilinear conductor, located in parallels to the test line. The length and distance of this conductor from the

line will be selected in such a way as to ensure a normal relationship of emf's in the antenna and loop. For feed of the antenna circuit it is also possible to use a voltage divider (Fig. 9.3).

First of all it is necessary to select such a resistance R that in the line there is established a traveling wave. Wave impedance of a single-wire line with diameter 2r at distance d from the conducting plane is equal to

$$\rho = 138 \lg \frac{2d}{r}. \tag{9.1}$$

By this formula there can be found the approximate value of resistance $R = \rho$. Traveling wave conditions in the line are verified by one of the known methods. In this case it is convenient to use the fact that impedance of a line, loaded on wave impedance, is equal to the wave impedance. Due to this, connection to the generator of standard signals of a line, loaded on resistance R, if $R = \rho$, will influence the generator the same as connection of the actual resistance R (will cause the same decrease of its output current). By several tests it is possible to definitize magnitude R, initially found by the formula (9.1). Traveling wave conditions must be verified in the whole range of frequencies of the direction finder.

Line calibration, i.e., determination of the field strength corresponding to the given output voltage of the generator, is produced by a comparator. The antenna of the comparator should be loop-type and of approximately the same dimensions as the loop of the direction finder.

If generator voltage is U, and field strength is E, then $k_{sf} = \frac{E}{U}$ is called the scaling factor, determination of which is the purpose of calibration.

Calibration should be performed at several frequencies within the frequency range of the direction finder. Independence of the scaling factor from frequency is confirmation of the fact that in the line there have been established traveling wave conditions.

It is necessary also to produce calibration for different distances of the center of the loop from the line.

If there is no comparator, calibration can be produced by a loop, whose geometric dimensions are known exactly. The emf on terminals of the loop should be measured by a voltmeter with a very large input impedance. As such voltmeter we use receiver with supply of voltage to the cathode grid of the first tube. The receiver is calibrated from a generator of standard signals.

If maximum emf in the loop is E_{max}, and voltage from the generator is U,

$$k_{rn} = \frac{E_{max}}{h_e U}, \qquad (9.2)$$

where h_e is the calculated effective height of the loop.

For selection of an auxiliary antenna or specifications of the divider feeding the antenna circuit, we should know the effective height of the open antenna of the direction finder h_a.

The emf introduced into the antenna circuit of the direction finder in real conditions is equal to

$$E_a = E h_a.$$

When testing under a line with the help of a divider this emf is equal to

$$E_a = U \frac{C_1}{C_1 + C_2}.$$

From this we find

$$\frac{C_1}{C_1 + C_2} = \frac{E}{U} h_a = k_{rn} h_a. \qquad (9.3)$$

The sum of capacitances $C_1 + C_2$ should be equal to the capacitance of the antenna C_a. Formula (9.3) gives the possibility of determining C_1 and C_2:

$$C_1 = C_a h_a k_{rn}, \qquad (9.4)$$

$$C_2 = C_a (1 - h_a k_{rn}). \qquad (9.5)$$

Testing under a line permits determining the same parameters of a direction finder as testing with the help of an equivalent antenna. Furthermore, testing under a line permits checking the sharpness of minima and the magnitude of errors depending upon frequency, field strength and other factors.

For checking selectivity by the two-signal method there is stretched a second line, perpendicular to the first and fed by a separate generator. Frequency and field strength of the disturbing radio station are established on this second generator.

It is necessary to note that neither the first nor the second method of laboratory testing corresponds fully to real conditions of work and, therefore, they can give results, differing from results of tests in operational conditions. Nonetheless, laboratory tests are very desirable, since thanks to the easy of shifting frequency, change of amplitude of the fed voltage, etc., tests can be conducted more widely and deeply than during tests on real work. Here, there can be revealed defects which would be passed over during performance tests.

Of the two methods described, obviously, the second corresponds more closely to real conditions of work of the direction finder, but carrying it out is somewhat more complicated than for the first.

9.2. Laboratory Tests of Direction Finders of a Goniometric System

Tests of Loops

Besides normal checking (determination of inductance, self-capacitance, damping, and so forth) for loops of goniometric systems it is very important to check the magnitude of mutual inductance between them. Absence of mutual inductance simultaneously confirms their mutual perpendicularity. From smallness of permissible magnitude of mutual inductance (permissible coupling coefficient is of the order of 0.3-0.4%) normal bridge and resonance methods are insufficiently exact.

A measuring circuit, permitting a reading, with the required degree of accuracy, is presented in Fig. 9.4. B is a variometer with very small inductances of windings (considerably smaller than inductance of loops), but with a fairly strong maximum coupling between them (K = 0.4 to 0.6). The high coupling coefficient permits sufficiently accurate calibration of the variometer. One of the windings of the variometer, series-connected with one of the loops, is fed from the generator; the other winding of the variometer and the second loop are also coupled in series and are joined to the cathode grid of the first tube of the receiver. Audibility on the receiver output turns into zero when the coefficient of mutual inductance of the variometer is selected equal to the coefficient of mutual inductance of the loops.

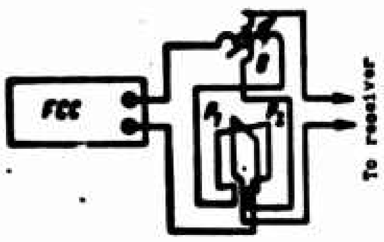

Fig. 9.4. Measuring circuit of small mutual inductance.

The generator and receiver, and also the variometer must be shielded, and all wiring is carried out in such a way as to exclude spurious couplings between circuits of the two loops.

Testing of the Goniometer

In the goniometer all its electrical parameters — inductances and distributed capacitances of all coils and maximum coupling coefficient between each of the field and the searcher coils — are to be checked. It is necessary also to check the coefficient of mutual inductance between the two field coils. This measurement can be made by the same scheme as analogous measurement for loops.

The most important test of a goniometer is determination of the error curve. Measurement of erros can be taken at high and low frequencies.

For checking at high frequency we compare the tested goniometer with a standard one, connecting them as shown in Fig. 9.5. Let the rotor of the standard goniometer turn about the first coil of the stator at angle θ_{st}. Assuming that the standard goniometer is absolutely exact we can present the emf's induced in stator coils in the form

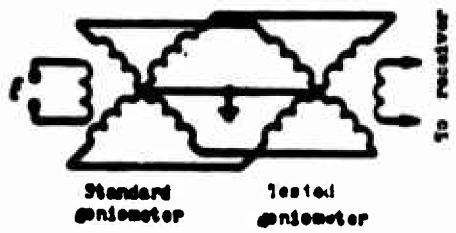

Fig. 9.5. Comparison of goniometer with a standard one.

$$E_1 = j\frac{E\omega M_1}{Z_1}\cos\theta_{st},$$
$$E_2 = j\frac{E\omega M_1}{Z_1}\sin\theta_{st},$$

where M_1 — maximum mutual inductance;

Z_1 — impedance of rotor of standard goniometer;

E — feed voltage.

Currents in stator coils will be

$$I_1 = jE\frac{\omega M_1}{Z_1(Z_{st}+Z_{x1})}\cos\theta_{st},$$
$$I_2 = jE\frac{\omega M_1}{Z_1(Z_{st}+Z_{x2})}\sin\theta_{st},$$

where Z_{st}, Z_{x1} and Z_{x2} — impedances of stator coils of the standard and investigated goniometers.

Normally impedances of two stator coils are equal to one another, i.e.,

$$Z_{x1} = Z_{x2} = Z_x.$$

If the searcher of the tested goniometer is turned an angle θ_x, then the emf induced in it will be

$$E_0 = -E\frac{\omega^2 M_1 M_2}{(Z_{st}+Z_x)Z_1}\cos(\theta_{st}-\theta_x),$$

where M_2 — maximum mutual inductance between field and searcher coils of the tested goniometer.

This emf turns into zero when $\theta_x = \theta_{st} + 90^\circ$. Thus, setting the rotor of the standard goniometer at some angle θ_{st}, we should obtain disappearance of audibility upon setting the rotor of the tested goniometer at an angle $\theta_{st} + 90^\circ$. The difference between this angle and the angle of setting, at which we obtain real disappearance of audibility, directly gives error of the goniometer. In an analogous way we can test a goniometer with three or four field coils.

The circuit of other method of testing at high frequency is presented in Fig. 9.6. If we select resistance so that $R_4 \ll \omega L_f$, where L_f — inductance of field coil,

R_4 — impedance of the divider, voltage division depends exclusively on the magnitude of resistances. Thus, voltage on one of the field coils will be

$$E_1 = E\frac{R_1}{R_4},$$

and on the other field coil,

$$E_3 = E\frac{R_3}{R_4}$$

etc., where R_1, R_2, ..., R_4 — resistance from beginning of divider to the corresponding tap.

Analogously to the preceding we find the emf in the searcher coil, assuming the goniometer is free from errors. Thus, with coupling of field coils into taps R_1 and R_3 we obtain

$$E_s = E_1\frac{M}{L_s}\cos\theta + E_3\frac{M}{L_s}\sin\theta =$$
$$= E\frac{M}{L_s}\left(\frac{R_1}{R_4}\cos\theta + \frac{R_3}{R_4}\sin\theta\right),$$

where M — maximum mutual inductance of the field and searcher coils of the goniometer.

The emf turns into zero when $\tan\theta = -\frac{R_1}{R_3}$.

If the goniometer gives error, then disappearance of audibility will occur at another angle Φ. Error of the goniometer will be equal to

$$\Delta = \Phi - \theta.$$

Thus, attaching the ends of field coils to various terminals of the divider and determining the position of the searcher corresponding to vanishing of audibility in the telephone, we can determine the error of the goniometer at different angles. It is possible to have a comparatively small number of taps in the divider (3-4), in order to obtain sufficiently closely located points for construction of the error curve.

Shielding of the generator, receiver, and divider, thoroughness of location of wiring in this method are as necessary as in the method of a standard goniometer.

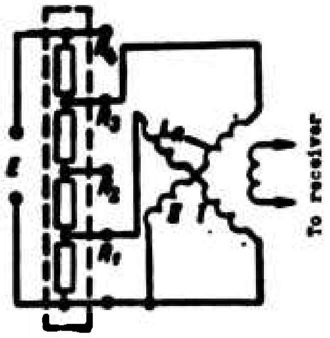

Fig. 9.6. Testing a goniometer by a divider.

The divider itself should be made inductionless and non-capacitive, possess a small skin effect, which is necessary for preservation of constancy of the ratio of resistances during change of frequency. One should make it with the same care as, e.g., attenuators of generators of standard signals.

The circuit for checking a goniometer at low frequency is shown in Fig. 9.7. In it R_1 and R_2 —

precision resistance boxes; L_1 and L_2 — field coils of goniometer; L_3 — its searcher coil; T — telephone (low-resistance).

The circuit is fed from an af generator G. Resistances R_1 and R_2 should be taken considerably larger than induced resistance of field coils with frequency of measurement ω, i.e., $R_1 \gg \omega L_1$ and $R_2 \gg \omega L_2$. In this case currents I_1 and I_2 are determined by equalities

$$I_1 = \frac{E}{R_1} \text{ and } I_2 = \frac{E}{R_2}.$$

If the goniometer was made absolutely exactly, the emf induced in the searcher coil would be

$$E_3 = I_1 \omega M \sin\theta \pm I_2 \omega M \cos\theta.$$

Rotating the searcher coil until audibility disappears in the telephone, we obtain angle θ from equation

$$I_1 \omega M \sin\theta \pm I_2 \omega M \cos\theta = 0$$

or

$$\operatorname{tg}\theta = \pm \frac{R_1}{R_2}. \qquad (9.6)$$

If the goniometer has error, audibility will disappear at another angle $\Phi = \theta + \Delta$, where Δ — degree of error. The method of checking consists in establishing ratio $\frac{R_1}{R_2}$ conforming to angles $\theta = 0°, 10°, 20°$, etc., and determining angle Φ at which sound disappears in the telephone. Difference

$$\Phi - \operatorname{arc\,tg}\frac{R_1}{R_2} = \Delta$$

directly gives error of the goniometer. To each ratio $\frac{R_1}{R_2}$ there correspond two angles differing approximately by 180°, at which audibility vanishes.

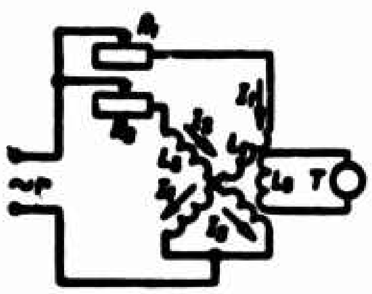

Fig. 9.7. Circuit for checking of goniometer at low frequency.

Thus, the goniometer is checked from 0° to 90° and from 180° to 270°. To check the second half of the dial the ends of one of the field coils are connected, which corresponds to a change of sign in formula (9.6). During work it is necessary to watch to see that the generator does not directly influence the searcher coil, and that current in the telephone does not influence the field coils.

Analogous circuits can be easily composed for testing multiwinding goniometers.

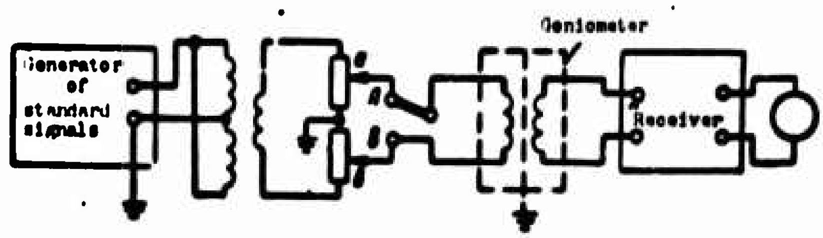

Fig. 9.8. Diagram for checking symmetry of goniometer.

So that in the goniometric system there is no antenna effect, it is necessary to ensure complete symmetry of field coils of the direction finder. Check of symmetry of the goniometer can be performed by the circuit in Fig. 9.8. Voltage from the generator of standard signals is brought to the field coil through a symmetric transformer (see § 4.3) and a potentiometric circuit of resistances.

The searcher coil is connected to the receiver. In switch position A voltage acts between ends of the field coil, which corresponds to reception of a two-phase wave. In switch position B the emf acts between both ends of the field coil and the "ground" (i.e., the frame of the goniometer, cathode of the first tube of the receiver and its frame), which corresponds to reception of a single-phase wave. A completely symmetric goniometer in the second switch position will not transmit voltage to the searcher coil.

In practice measurement is performed in the following way. Setting the switch in position A, tuning the receiver and turning the searcher coil to the position of maximum coupling with the tested field coil, we regulate the voltage of the generator of standard signals to obtain a conveniently read receiver output voltage U. Let us assume that here the voltage of the generator of standard signals is equal to E_1.

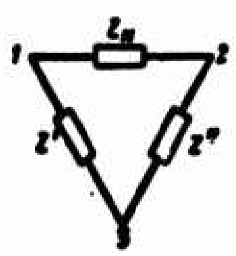

Fig. 9.9. Circuit of asymmetric loading.

Then we shift the switch to position B, increase output voltage of the generator of standard signals and turn the searcher coil to obtain maximum receiver output voltage. Let us assume that voltage of the generator of standard signals, necessary for production of the same receiver output voltage U, in this case is equal to E_2. Then the relative degree of asymmetry of the goniometer is characterized by ratio $\frac{E_1}{E_2}$.

In carrying out tests it is necessary to ensure symmetry of the transformer, equality of potentials at points a and b, and also to avoid any asymmetry of the

goniometer (for instance, because of asymmetric position of wires to the switch).

Measurement of asymmetry by another method is carried out with the help of an hf resistance bridge. Asymmetry is caused by unequalness of capacitive or in general, any impedances between terminals 1 and 2 of load Z_{load} (in this case the goniometer) and the ground. In Fig. 9.9 these impedances are designated Z' and Z''. The asymmetry parameter is equal to

$$k_{as} = \operatorname{mod} \frac{Z' - Z''}{Z' + Z''}.$$

We take three measurements of admittances:

1) between point 1 and grounded point 2 (Y_1);
2) between grounded point 1 and point 2 (Y_2);
3) between short-circuited terminals 1 and 2 and ground (Y_3);

$$Y_1 = \frac{Z' + Z_n}{Z' Z_n}; \quad Y_2 = \frac{Z'' + Z_n}{Z'' Z_n}; \quad Y_3 = \frac{Z' + Z''}{Z' Z''}.$$

It is easy to see that

$$k_{as} = \operatorname{mod} \frac{Y_2 - Y_1}{Y_3}.$$

Both methods of measurement of asymmetry are applicable also to measurement of asymmetry of the input of the receiver and of other elements.

Test of a Radio Direction Finder as a Whole

To test a loop radio direction finder of a goniometric system in laboratory conditions there can be employed the same two methods as for testing a direction finders with a rotating loop, i.e., testing with an equivalent and testing with the help of a line. The equivalent presented in Fig. 9.1 gives the possibility of feeding emf only to one of the field coils. The remaining field coils of the goniometer should be closed to the same equivalents with closed input terminals.

When testing by the two-signal method there can be used a second field coil, to which there is fed an emf from a second generator through an antenna equivalent.

In the case of an external system of two spaced antennas it is also possible to compose an equivalent. Its circuit is presented in Fig. 9.10. Here C_a, C_{fd}, C, C_{fair} – capacitances of spaced antennas, feeder, auxiliary antenna and its fair-lead; L and L_a – inductances of the auxiliary and the spaced antennas. Selection of magnitudes R, R_a and M is analogous to the preceding case. It should be stressed that testing by an equivalent has meaning only for those systems, for which natural waves of antennas considerably differ from working waves.

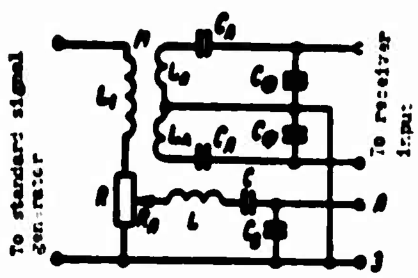

Fig. 9.10. Circuit of the equivalent for a system of speed antennas.

9.3. Laboratory Tests of Radio Direction Finders with Wide Antenna Spacing

Separate component parts of the radio direction finder (hf transformers, time delay line, switching circuits, indicators, etc.) are checked by usual methods.

The antenna system of a radio direction finder with wide spacing antennas consists of a large number of antennas, in which there are induced emf's of identical amplitude, but with different phases in accordance with geometric location of the antennas (§ 3.11). Antennas are connected to an antenna switch or a reception-indicator.

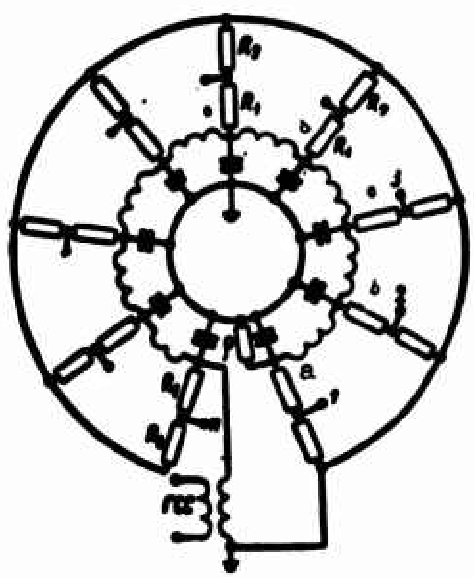

Fig. 9.11. Circuit of the equivalent of a circular antenna system with wide spacing.

During laboratory tests of a radio direction finder it is necessary to be able to introduce to inputs of the antenna switch (or reception-indicator) voltage of identical amplitude, the phase of which varies by a given law. For this there is used a special antenna equivalent. In Fig. 9.11 there is presented the circuit of the equivalent for laboratory testing of a radio direction finder with a circular antenna system. It consists of a natural or artificial long line, fed by a generator of standard signals and loaded on an impedance, equal to wave impedance. The section of long line is designed in such a manner that on terminals of the long line of the equivalent a, b, c, etc., voltages have identical amplitudes and phases, equal to phases of the emf of corresponding antennas.

Phases of voltages are calculated for the case when there is produced reception of a radio station from a definite direction.

Between terminals of the long line and the ground there are coupled resistance R_1-R_2 such magnitude that $R_1 \gg R_2$ and $R_2 = \rho_{fd}$, - where ρ_{fd} wave impedance of feeders leading into the antenna switch (matched loading of feeders from the antennas is assumed).

Thus, on the output terminals of the equivalent 1, 2, 3, ..., n there are voltages of identical amplitude with phases, corresponding to phases of emf antennas. Output resistances on these terminals are equal to ρ_{fd}. Application of decoupling resistances R_1 removes influence of loads of the antenna switch on the amplitude and phase of voltages at points 1, 2, 3, ..., n.

The antenna equivalent permits checking the overall efficiency of equipment, determining instrument accuracy for fixed directions and sensitivity. Instrument accuracy is determined connecting the output of the equivalent 1, 2, 3, ..., n first to terminals 1', 2', 3', ..., n' of the antenna switch. Here, on the antenna switch during direction finding there should be read an angle, corresponding to that bearing, for which the long line is calculated. Then the output of equivalent 1, 2, 3, ..., n are switched to terminals 2', 3', ..., n', 1' of the antenna switch, 3', 4', ..., n', 1', 2', etc. Each switching corresponds to displacement of the direction of bearing an angle, equal to the angle between the antennas. The difference between readings on the bearing indicator of the radio direction finder and calculated bearings corresponds to instrument errors. To determine sensitivity it is necessary to preliminarily find coefficient k of transmission of voltage from input terminals of the equivalent to its output terminals 1, 2, 3, ..., n with connected loads.

If to the equivalent's input there is fed voltage U, then $E = \frac{U}{kh_{eff}}$, where h_{eff} is the effective height of the antenna.

With an unmatched loading of feeders instead of ρ_{fd} it is necessary to couple in at each frequency its own Z_a, corresponding to input impedance of the antenna and feeder together.

By this method we also determine the directivity pattern of the antenna system.

9.4. Tests of Direction Finders in Real Conditions of Work

When testing a direction finder in the place of installation it is necessary to check separate parts of the antenna-feeder device (single angennas, feeders, etc.) and correctness of their geometric location. Tests are performed by methods, described in [9.3].

Tests of a radio direction finder have the goal of determining: instrument error of the direction finder, magnitude and nature of local errors, general accuracy of the direction finder, general sensitivity of the direction finder, the characteristic and coefficient of directivity of its antenna system.

Determining Instrument Error of a Radio Direction Finder

It is not possible to determine instrument error for all systems of direction finders. Thus, direct determination of instrument errors for direction finders with a fixed outdoor system is impossible, if the latter is too bulky. In these cases it is necessary to be limited to analysis of separate sources of instrument error on the basis of laboratory tests and tests, which are described in the following point.

Instrument error is most exactly and simply determined for goniometric direction finders, for which structure and dimensions of the outdoor equipment are such as permit rotation of it in the process of testing. For this purpose the external device of the direction finder is set on a special machine, permitting us to turn it at known angles. Tuning to some station, by rotation of the goniometer we find its bearing. We then turn the outdoor system a certain angle (for instance, 10-15°) and repeat fixing. The new reading on the goniometer should differ from the first by the angle of rotation of the outdoor system. Performing such tests for several angles from 0° to 360° and at various frequencies, we can obtain a sufficiently full judgement of instrument accuracy of the direction finder.

Special difficulties are presented by tests of direction finders with calculation of polarization errors. Thus, to determine standard polarization error one should place the direction finder in an electromagnetic field with a known slope of the wave front and angle of polarization. For creation of such a field a local generator is placed at a considerable height (on a mast, ballon, etc.) and is equipped with a radiating dipole, which is set at such an angle as creates a field with the necessary turn of the plane of polarization.

Distance from the direction finder to the generator should be sufficiently great. For direction finders of short waves this distance is practically of the order of 100 m or more. To create an angle of incidence of 45°, corresponding to conditions of test of the error of a standard wave, height of lift of the generator should also be near 100 m. This causes evident practical difficulties, because of which in most cases we are limited to smaller height of rise of the emitter. So that polarization error is not small, the angle of rotation of the plane of polarization γ is made greater than 45°.

If the angle of inclination of the wave front is very small, to satisfy the shown condition angle γ should be close to 90°. Inconvenience of such a condition of tests is the small magnitude of the vertical component of field strength and, consequently, the weak reception power.

To produce reliable results of measurement it is necessary to ensure strict symmetry of the generator, since the presence of a single-cycle wave in a radiating dipole does not permit establishing the exact relationship between magnitudes of vertical and horizontal components of the field. The direction finder should be located in a plane, perpendicular to the plane containing the radiating dipole. Otherwise the ratio of horizontal and vertical components of field strength is not equal to the tangent of the angle of rotation of the dipole. Such a phenomenon does not occur with a radiating loop; therefore in installations for checking polarization errors they chiefly apply a loop as the emitter.

Determining the Magnitude and Nature of Local Errors

To determine errors of the direction finder the generator shifts around it. Direction finding is produced at different positions of the generator, and results are compared with true angles. These angles, depending upon the circumstances, are determined either by visual direction finding, or on the map by known positions of the mobile generator.

The mobile generator is carried, trucked or is placed on a ship, aircraft, etc. If the direction finder is placed on a mobile object (ship, aircraft), one can determine error by a fixed source of radiation by means of shifting the direction finder itself.

The distance from the direction finder to the source of radiation should be sufficiently large, in order to consider the field near the direction finder as the field of radiation, i.e., the distance should exceed the wavelength. Furthermore, the distance should be sufficient, that there is formed an approximately flat wave front:

$$r > \frac{2(2b)^2}{\lambda},$$

where 2b — antenna aperture (spacing of vertical antennas).

The generator should be located in an open locality far from objects which could create a field of secondary radiation.

The error curve, found by the shown method, contains both instrument errors of the radio direction finder, and also locality errors. In order to separate errors, it is necessary to determine instrument errors by other methods (see the first point of this paragraph) or conduct inspection of local errors by a radio direction finder, whose instrument errors are small and are known. In separate cases judgement about whether error is local or instrument is facilitated by consideration of the dependence

of error on the distance to the radiator, i.e., from shift of the heterodyne with constant azimuth. Instrument error does not depend on range, and local error changes with change of range.

With a separate re-emitter the dependence of error, caused by it, on the distance between the heterodyne and direction finder has a regular sinusoidal nature, which facilitates detection of such error.

The general character of the error curve also depends on the distance between the direction finder and generator (§ 10.5). When this distance is small (for instance, 100-300 m), influence of locale and surroundings is not transmitted completely in the obtained error curve, since relative magnitudes and phases of field strength of reverse emitters differ from corresponding magnitudes obtained under the influence of a wave from a very distant source.

It is possible to recommend such tests mainly for checking instrument errors under the condition that local errors are minute.

With large distances (for instance, 3-5 km) conditions of testing are closer to real conditions of work of the radio direction finder and more fully reflect local errors, including and the influence of distant surroundings. These tests should be conducted during the introduction of the radio direction finder into service to judge its general accuracy.

It is necessary to note that the curve of local errors, taken with the help of a generator located on the surface of the earth, can not preserve its form during reception of reflected rays, since reverse radiations, provoking local errors, change their character under the influence of an abnormally-polarized field.

Determining General Accuracy of a Radio Direction Finder

Because of the above-indicated difficulties separate determination of instrument errors, although of considerable interest, cannot completely characterize general accuracy of a radio direction finder. On the other hand, determination only of the general accuracy of a direction finder, the most important of its operational characteristics, says little to the designer, since he cannot find which part of errors can be eliminated by improving the design of the instrument and which part can be eliminated by improving its location. Therefore, whenever possible, one should make a full investigation of the direction finder, determining both instrument and local errors, and also the general accuracy of the direction finder.

To determine general accuracy of a direction finder we perform direction finding

of different known objects, located as far as possible in different directions, at different distances and working on different waves.

Comparing the found radio bearing with the true one, determined by calculation or plotting on a map, we find error. For the characteristic of general accuracy of a direction finder we find the mean error. According to the theory of errors it is most correct to take for mean error the mean quadratic error, but for simplification of calculations we frequently limit ourselves to finding the arithmetic mean error (without taking into account sign).

For radio direction finders, working on short and medium waves, during direction finding at night scattering of errors is so great that their average magnitude insufficiently characterizes work of the direction finder. Methods of analysis of observations are presented in Chapter 11.

Determining General Sensitivity of a Radio Direction Finder

To determine general sensitivity of a direction finder we simultaneously perform direction finding and determination of field strength by a comparator. During direction finding by hearing we note the angle of silence. During visual direction finding we note the line width of the image of bearing or the magnitude of oscillations of the pointer of the indicator from the influence of noises.

On ultrashort, medium and long waves sensitivity can be determined by means of observation of different remote radio stations. On short waves observation of remote stations because of the influence of fade-outs leads to very inaccurate determinations of sensitivity. Therefore, on short waves sensitivity is better determined by using a local generator, removed a certain distance from the direction finder.

Field strength in all cases is measured by a comparator. If there is no comparator, sensitivity can also be determined by the local generator, equipped with an ammeter in the antenna, whose virtual height is known. In this case field strength near the generator is calculated by the corresponding formulas. This method is less exact than direct comparison.

Sensitivity of a radio direction finder may also be found by means of calculation, if there are determined the sensitivity of the receiving equipment and the virtual height of the antenna system (see §§ 2.7-2.11).

One can determine virtual height in the following manner (Fig. 9.12): at a certain distance d from direction finder DF there is located emitter G. Exactly the same distance d from the latter there is placed comparator C, so that field strength

for the direction finder and for the comparator is identical. The distance between the direction finder and the comparator should be such as to exclude their interaction. Field strength created by the emitter is measured by the comparator. Its value E_0 should substantially exceed the level of field strength of the external interferences. The direction finder is tuned to the wave of the emitter, and its antenna system or goniometer is set in a position, corresponding to maximum reception power. Voltage on its output is measured by a voltmeter. By gain control instruments output voltage is set such that work of the receiver in the linear region is ensured. To ensure linearity there will also be turned off the automatic gain control.

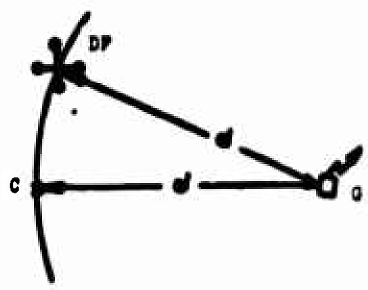

Fig. 9.12. Diagram of location of instruments during measurement of virtual height.

Then we disconnect the antenna system from the receiver and connect to it a generator of standard signals through antenna equivalent. In the goniometer system all non-operating windings of the goniometer should here be loaded on antenna equivalents Eq (Fig. 9.13). Without touching receiver controls, we adjust the voltage of the generator of standard signals to obtain on the receiver output the same voltage as was obtained earlier from the external emitter. Obviously, in this case voltage in the antenna circuit E_g from the generator of standard signals is equal to the voltage in the antenna circuit E_0H_a, which the external emitter created.

On the basis of this we can find virtual height as $H_a = \frac{E_g}{E_0}$*.

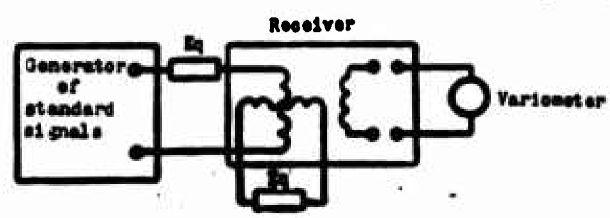

Fig. 9.13. Circuit diagram of receiver.

Determining the Directivity Pattern and the Directivity Factor

To determine the directivity pattern of the antenna system of the direction finder in the horizontal plane it is necessary to determine the dependence of output voltage of receiver on angle, which the direction of propagation of the wave forms

*This formula is valid when resistance of the equivalent is equal to resistance of the antenna.

with the plane of the antenna system. In a system with a turning antenna this angle can be changed by rotation of the antenna system itself. The radiation pattern of a radio direction finder with fixed antennas can be found by rotating the device intended for rotating the radiation pattern (goniometer, commutator) and noting the dependence of output voltage on the angle of rotation of the goniometer or commutator. Another method of finding the radiation pattern consists moving the sender around a fixed antenna system on a circumference, whose center coincides with the center of the antenna system.

One should equip the sender with an ammeter for measuring in it the current, which should be maintained strictly constant. To ensure great accuracy it is possible to recommend monitoring constancy of field strength in the direction finder by a comparator.

Angular shift of the sender is measured by a compass, theodolite, or another such instrument.

Output voltage of the receiver is measured by a voltmeter. It is necessary beforehand to check the range of voltages, in which output voltage linearly depends on input, and during the whole test voltage should not leave the linear region.

Dependency $\frac{U_{out}}{U_{max.out}} = F(\theta)$, presented graphically in polar or cartesian coordinates, is the sought directivity pattern. The directivity factor in the horizontal plane is determined graphically from the directivity pattern:

$$D_{\text{г}} = \frac{2\pi}{\int\limits_0^{2\pi} \rho^2 d\theta},$$

where

$$\rho = \frac{U_{\text{вых}}}{U_{\text{вых макс}}} = F(\theta).$$

Thus, to determine the directivity factor it is necessary to construct the curve of the dependence of ρ^2 on angle θ and by planimetry find its area $\int_0^{2\pi} \rho^2 d\theta$. Number 2π, divided by the found area, gives the directivity factor.

If field strength is measured by a comparator, then before completing the shown calculations it is necessary to introduce a correction in values of measured output voltages, determining magnitude

$$U'_{\text{вых}} = U_{\text{вых}} \frac{E_{\text{макс}}}{E},$$

where E_{max} — maximum field strength;

E — field strength, measured at the same position of generator, at which U_{out} is measured.

During construction of the radiation pattern we use values of U'_{out} and $U'_{max.out}$.

Measurement of directivity patterns in the vertical plane causes considerable technical difficulties; with this purpose it is possible to use aircraft and helicopters [9.3].

MT-65-58
Principles of Radio Direction Finding,
"Soviet Radio" Publishing House,
Moscow, 1964.
Pages: 542-560.

CHAPTER 10

DIFFERENT APPLICATIONS OF RADIO DIRECTION FINDERS

List of Designations Appearing in Cyrillic

и = i = [definition undetermined]
макс = max = maximum
пр = long = longitudinal
р = loop
П = RDF = Radio Directional Finder
п = trans = transverse

Below are certain practical instructions on selecting the site, installation, and adjustment of ship, aircraft and ground radio direction finders. In this chapter we used materials of published manuals and recommendations [1.10, 10.1, 10.6, 10.7].

10.1. Ship Radio Direction Finder

Selection of Site

For installation of the antenna array of a ship radio direction finder one should select the place, most removed from metallic parts of the ship. Therefore, the antenna array should be assembled as high as possible above the hull of the ship and as far as possible from stacks, masts, antennas and metal superstructures.

To indicate beforehand the best place is difficult. At medium waves the ships hull usually is the main influence. Knowing dimensions of the ship, it is possible by the formulas (5.58), (5.60) to approximately calculate deviation, caused by the hull when placing the antenna array directly on the deck or on a mast.

On short waves of greatest influence are antenna-like objects (masts, stacks, and so forth), tuned in resonance with the frequency of direction finding, when on their length there is laid-off a quarter of the wavelength or three quarters of the wavelength (Fig. 5.14).

Thus, 30 m mast creates the greatest deviation at frequency $f = 2.5$ Mc; the captain's bridge 15 m high, at frequency $f = 5$ Mc; a 11 m stack, at frequency $f = 7$ Mc. Range of frequencies at which such objects act depends on their transverse dimension. An antenna made from a conductor practically has effect in a range of ±5% on both sides of resonance frequency; a stack, in range ±10%; a bridge, in range approximately ±30%, etc.

On ultrashort waves large structures shield reception, and direction finding becomes impossible.

It is necessary before inspection on a ship to consider on the plans of equipment of ship the most convenient places and to calculate for them on the basis of materials of Chapter 5 maximum deviations.

Antenna systems with spaced loops or with spacing of antennas greater than λ, because of their somewhat sharper radiation pattern, are less subject to the influence of fields of reverse radiation. An antenna array with spaced antennas with small spacing reverse emitters is influenced to a still larger degree than a loop array, because of stronger action of the electrostatic fields on vertical antennas.

After places for mounting of the antenna system are noted (usually several), it is advisable to test them with a portable radio direction finder, if possible, and select the best place. To find the fitness of the place we take the curve of deviation along by one of the methods shown below. The best place will be the one, for which maximum deviation is less in the working range of frequencies of the radio direction finder. On medium waves the curve of deviation should be very close to quadratic (quadratic deviation is easy to compensate) and vary little with frequency.

On short and ultrashort waves deviation should vary smoothly with change of relative bearing of the station and with change of frequency. Furthermore, there should be obtained complete angles of silence for bearings of all directions 0-360° in a sound radio direction finder, small ellipses of images (not more than 15%) in an automatic two-channel radio direction finder, and precise readings of bearing in other systems of direction finders.

Finding of the curve of deviation and check of the quality of direction finding should be done at sea. As long as ships are in the dockyard, it is impossible to do these jobs, since around ship there usually are many foreign objects, creating additional errors.

The place for mounting receiver equipment should be convenient for work. The length of the high-frequency cable required for connection of the antenna array (of goniometric or other type) with the receiver should be as small as possible. The receiver equipment of a radio direction finder should be installed in the chart house, since the navigatory uses the radio direction finder.

Mounting of the Antenna Array of a Radio Direction Finder

Let us consider cases, when in the ship radio direction finder as the antenna array there are applied a rotatable loop, a goniometric system of two

mutually-perpendicular loops, a rotatable system of spaced loops, and a system with spaced antennas.

The antenna array is installed in such a manner that its axis of symmetry lies in the diametrical plane of the ship; otherwise there may appear a coefficient of constant deviation A, varying with frequency. Such deviation is difficult to compensate.

An auxiliary antenna, if it is not provided in the construction of the antenna array, is taken as far as possible vertical and located in direct proximity to the directional antenna.

In a rotatable system with reading by the minimum the dial is oriented in such a manner that it reads angle 90-270° when the plane of the system coincides with the diametrical plane of ship, or 0-180° when the plane of the system is perpendicular to the diametrical plane. In a goniometric system one loop or pair of antennas are usually mounted in the diametrical plane; the second is mounted across the ship. With unequal dimensions of loops (for a medium-wave radio direction finder) the smaller loop is installed along the longitudinal axis, thanks to which quarter deviation partially is compensated.

For a goniometric radio direction finder it is necessary to check correctness of connection of the ends of field coils of the goniometer to the loop device and of ends of the searcher coil to the receiver.

For this we listen to and fix any radio station, when to goniometer there is joined only one longitudinal loop (transverse is disconnected); the bearing should be 0° or 180°. If instead of 0° or 180° the bearing is equal to 90° or 270°, then to the longitudinal loop we join the other field coil of the goniometer.

Further we fix a radio station, when one transverse loop is connected; bearing should be 90° or 270°.

Finally, we join both field coils and check matching of fields. With rotation of the ship counterclockwise the bearing should increase; if this is not so, one should exchange ends in any of the field coils.

Correctness of determination of direction is attained by true connection of the ends of searching coil. If direction is determined incorrectly, we switch the ends of the searcher coil.

In the same manner connection to the receiver-indicator of a two-channel radio direction finder is checked.

It is very important that near the outdoor equipment of a radio direction finder all touching metallic parts (for instance, bulkhead of the bridge, guard rail, struts, etc.) have good contacts with the hull; otherwise with variable contacts, there is obtained variable deviation. Furthermore, during disturbance of contacts there is audible a crackling in the telephone of the receiver.

Cables and other metallic rigging nearest to the antenna array should have a length, smaller than $1/4\ \lambda_{min}$. If the antenna array is mounted on a separate mast, the upper part of the mast should be free of yards standing out to the sides, and so forth. Guys for the mast should be symmetrically located about the antenna system.

Taking the Curve of Deviation of a Ship Radio Direction Finder

The bearing q of a radio station, lying at heading p to the longitudinal axis of the ship, under the influence of metallic objects located around the loop (antennas, guys, metal hull, and so forth), is incorrect.

Deviation f is equal to

$$f = p - q.$$

The formula for f gives the absolute value and sign of deviation. Deviation generally varies with heading and depends on the length of the fixed wave. After installation of a radio direction finder before using it, it is necessary to determine deviation for all directions from 0° to 360° and for needed waves (take the curve of deviation). During subsequent work on the radio direction finder we use curves of deviation to determine corrections to bearings.

The curve of deviation is taken chiefly by a visible radio station, transmitting certain signals for this purpose. We determine visually headings p to the working radio station related to the longitudinal axis of ship and simultaneously take readings on the radio direction finder, i.e., determine angles q. Knowing p and q, we calculate deviation f and construct the curve of deviation in the form of the dependence of f on q.

The curve of deviation of a ship radio direction finder can be found by the following methods.

1. We use work of the sender of a non-directional radio beacon or auxiliary ship. The ship with the radio direction finder turns every 10-15° near the beacon or auxiliary-ship. On every course there is determined visually the heading to the transmitter p and there is made a reading on the radio direction finder q. Deviation is defined as the difference between these readings.

Instead of lie on courses every 10-15°, it is possible to accomplish continuous circulation and to take visual readings and radio direction finder readings every 10-15°.

The distance between the radio direction finder and the sender is more than 2-3 λ, in order to be in the field of radiation of the sender.

2. The ship with the radio direction finder can be turned with the help of an auxiliary tug. Such a method is employed for large ships, whose own movement is costly.

3. The ship with the radio direction finder can stand at anchor, and the auxiliary ship with radio transmitter passes around it. Reading every 10-15° of movement of the auxiliary ship simultaneously visual headings p and radio bearings q, we calculate deviation f.

4. It is possible to take deviation by an invisible radio station. We calculate the bearing α from the point of taking the deviation to the radio station. To determine the true bearing every 10-15° of turn of the ship we determine radio bearings q and simultaneously compass courses (KK). Knowing deviation of the compass ΔK and declination ΔM, we calculate p and f:

$$p = \alpha - (KK + \Delta M + \Delta K) \quad . = p - q.$$

When taking the curve of deviation it is necessary that at a radius of at least one nautical mile there are no other ships or harbour structures.

The power of radio station from which we take the deviation should be sufficient so that readings are absolutely clear. With large range of waves of the direction finder we take deviation on several waves.

Deviation varies with change of the drought of the ship.

So that there is no error from parallax, the distance from the loop of the radio direction finder to instrument, on which we read the heading, should be not more than 1/192 of the distance between the direction finder and the transmitter.

If in the radio direction finder there is no means of compensation for deviation, after determination of deviation and construction of the curve of deviation we calculate coefficients of the expansion of the curve in a Fourier series.

Calculation of coefficients of deviation is reduced to replacement of integral expressions of coefficients of the Fourier series by the sum of ordinates of the curve of deviation at definite intervals of degrees. In Table 10.1 (see insert at the end of the book) there is given the form for calculating coefficients of deviation using ordinates every 15°.

After determining coefficients of deviation we inspect around the loop of the direction finder to see if they are removable factors, causing any abnormally great coefficient (see § 5.8). As a rule, after removal of factors causing deviation it is necessary to again take the curve of deviation.

On the basis of the obtained results for subsequent use we construct curves of deviation or compose tables of deviation for approximately every 5^{o} change of q.

If in the radio direction finder there is equipment to compensate for deviation, then we first determine deviation only on eight headings: 0, 45, 90, ..., 315^{o} at that frequency, at which compensation is provided. We calculate coefficients A, D, and E, which compensate. Then we take residual curves of deviation by one of described methods.

10.2. Radio Direction Finder on an Aircraft

On an aircraft radio direction finder besides normal requirements there is presented an additional one – strength of construction (loop, receiver, etc.) with minimum volume and weight. Such a requirement is set because equipment on an aircraft experiences strong shocks. For combat shocks the receiver is fastened to special shock absorbers.

Aircraft interferences, in two forms – acoustic and electrical – adversely affect actual sensitivity of a radio direction finder.

In light of the impairment of the actual sensitivity from acoustic interferences during sound direction finding, on aircraft they apply visual radio direction finders, most frequently radio compasses and radio compasses.

Basic electrical interferences are interference from ignition and interference from electric generators and motors of the aircraft. The most effective method of counteracting interferences of ignition is complete shielding of the ignition circuit. Other methods do not lead to complete freeing from interferences in the wide band width of the direction finder. Interferences from electrical generators and motors, mainly commutator noises, are cancelled by blocking their circuits with chokes and capacitances, and measures are also taken to decrease sparking.

In spite of all the measures against interferences, the level of noises on aircraft is greater than on earth, and, consequently, sensitivity of a radio direction finder is worse.

The aircraft radio direction finder is often used for flight on a radio station. In this case the rotatable loop is installed across the airplane fuselage (the

goniometer is set at 0°), and the course of aircraft is regulated in such a manner that the loop remained in the position of bearing (in telephones, zero audibility, or on the indicator of the semicompass, a zero reading).

With such use of a radio direction finder from wind there is created drift of the aircraft and time of flight is extended. By selection of a leading course, at which the resultant speed of the aircraft and wind coincide with the direction of flight, it is possible to achieve a direct line of flight; for this we rotate the loop a bit. Correct rotation of the loop can be judged by preservation of the compass reading, if we maintain the course from readings of the radio direction finder. With increase of flight speed the influence of wind drift decreases.

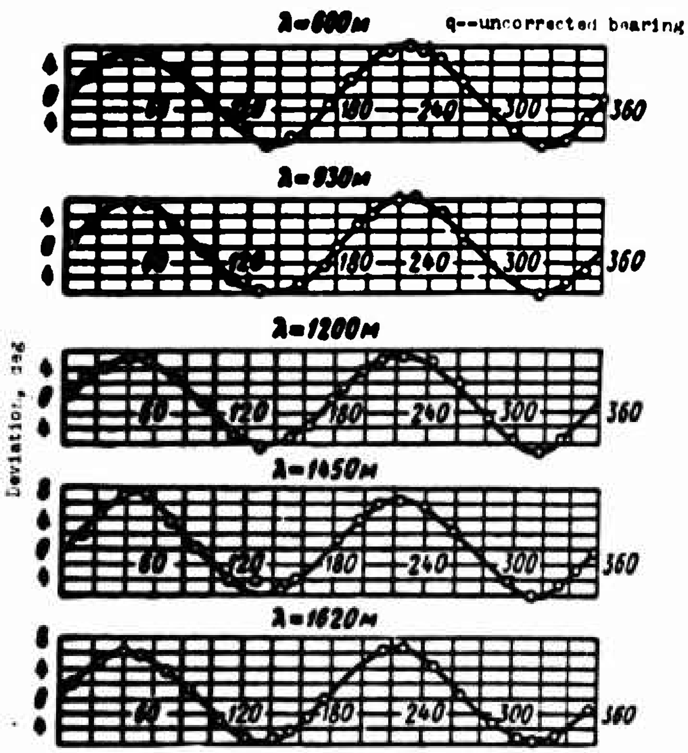

Fig. 10.1. Sample deviation curves.

Orientation and check-out of the mounting of the antenna array of the radio direction finder on an aircraft is carried out just as on a ship. Taking of the deviation of an aircraft radio direction finder in air presents difficulties. Usually we place the aircraft in flight position on the ground on a rotating circle and turn it at different angles to a local transmitter, located a distance of at least 2-3 λ from the aircraft [10.1]. By comparison of visual readings and radio bearings we determine deviation. The site where we take the deviation should be free of disturbing objects (wires, antennas, trees, etc.). The procedure for taking the deviation curve and analysis of results for an aircraft radio direction finder do not differ from the procedure and analysis for a ship radio direction finder.

Sample deviation curves of a medium-wave radio direction finder on an aircraft are given in Fig. 10.1.

10.3. Compensation of Deviation in a Radio Direction Finder with a Rotatable Loop

There exist several methods of compensation of deviation of a radio direction finder. They can be divided into two groups: methods of mechanical compensation of

deviation and; methods of electrical compensation of deviation.

In some installations both methods are used simultaneously.

Mechanical Methods of Compensating Deviation

Principle of work of a mechanical compensator is based on the fact that the dial (or dial indicator) on which the bearing is read fitted on the shaft (of the loop or goniometer) by an auxiliary device. This device creates displacement of the dial (or dial indicator) with respect to the loop or searcher coil of the goniometer an angle, equal to the deviation, and thus deviation is compensated.

In Fig. 10.2 is depicted a system of four levers a, b, c, and d, connected at points 1, 2, 3 by hinges. Between points 1 and 3 there acts a spring, thanks to which hinge 2 and the roller attached to it constantly press on disk L. Lever a is connected to the shaft of the loop (or searcher coil of the goniometer); lever b is connected to the dial indicator. With rotation of the loop shaft the dial indicator tracks the loop.

If disk L has the form of a circle, then the angle between the loop and the dial indicator remains constant, since their relative position does not change. Giving disk L the corresponding form, differing from a circle, it is possible so to move the dial indicator with respect to the loop (leading with an indentation and lagging with a projection) as to compensate any deviation of the radio direction finder.

Sometimes instead of disk L we employ flexible steel tape, whose form is regulated by levers. The roller goes inside tape. The spring draws points 1 and 3 together.

The mechanical compensator can compensate deviation of any law. However, it is advisable to compensate only even coefficients of deviation D, E and constant deviation A. Actually, if one were to compensate semicircular deviation (B and C), then, reading the bearing not from the correct side ($q \pm 180^{\circ}$), we make an error, equal to

$$2(B\sin q + C\cos q),$$

and with noncompensated semicircle of deviation, if we do not consider deviation, error is equal only to

$$B\sin q + C\cos q.$$

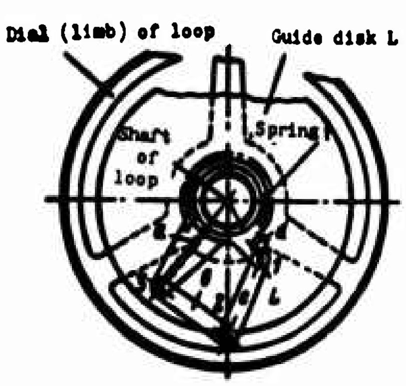

Fig. 10.2. Mechanical compensator of deviation.

The mechanical compensator cancels deviation on one wave. For other waves there are given tables and curves of residual deviation.

Electrical Compensation of Deviation by Installing a Loop

As was shown, of greatest importance in deviation of ship and aircraft radio direction finders, working on medium and long waves, is quadrant deviation. Therefore in practice we are limited to electrical compensation only of this deviation, although in principle we can compensate electrically for other components of deviation. We already saw that the body of a metallic ship (aircraft, dirigible) creates deviation with the same law, as a re-emitting loop. On the basis of this it is possible to compensate deviation caused by the metallic body by installation of an auxiliary frame.

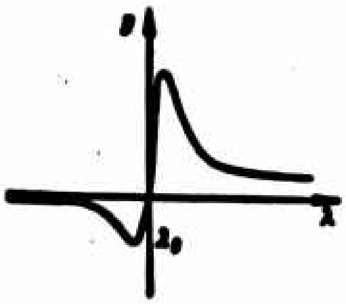

Fig. 10.3. Change of coefficient of deviation D with change of wavelength.

The body of a metallic ship (aircraft, dirigible) is equivalent to a closed loop with characteristic wavelength λ_0, approximately equal to twice the length of the body. The law of change of D with change of the received wave λ is shown in Fig. 10.3. Ideal compensation on all waves could have been achieved by construction around the direction finder loop of a closed loop with parameters, equal to parameters of the frame which is equivalent to the body. In practice, this is unrealizable, since we obtain very large dimensions of the compensating loop. It is possible to construct compensating loops of smaller dimensions, and couple them to the direction finder loop in such a manner as to compensate deviation, e.g., on long waves. Since the fixed wave will be much larger the characteristic wave of such an additional loop, the compensated deviation will almost not vary with change of the waves of direction finding. This will occur until the wave approaches the characteristic wave of the frame, equal to its perimeter. It turns out that by an additional loop, practically relizable, it is possible to compensate coefficient D, constant on the wave range.

For a great many aircraft and ships in their working range of direction finding coefficient D remains constant, and, consequently, for compensation it is possible to choose a compensating closed circuit.

According to formula (5.43) we had

$$D = \frac{m}{2+m},$$

where

$$\dot{m}=\frac{j\omega M_{m}h_{0\kappa}}{Z_{0\kappa}h_{g}};\ h_{0\kappa}=\frac{2\pi S_{2}}{\lambda};\ h_{g}=\frac{2\pi N S_{1}}{\lambda};$$
$$Z_{0\kappa}\approx j\omega L_{0\kappa}.$$

Given the area of compensating loop S_2, it is possible to calculate L_{01} and M_{max} and then m (N and S_1 – number of turns and the area of the direction finder loop – are known). From m we calculate coefficient D, compensated by the additional loop. Thus, by approximation it is possible to find the required compensating loop.

Sometimes for compensation they make the additional loop rigid, consisting of two spaced steel rings, fixed motionlessly, symmetrically to the shaft of the loop on both sides of it, along the longitudinal axis of the ship (aircraft).

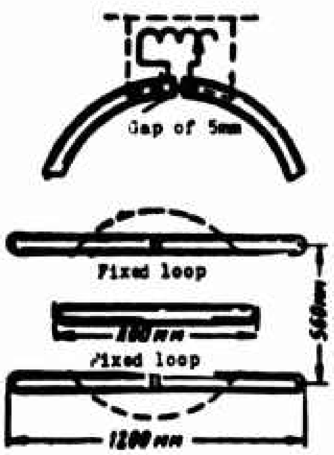

Fig. 10.4. Fixed compensating loops.

By changing the inductance of coil, which closes the rings (Fig. 10.4), it is possible to achieve compensation of the required value of coefficient D. The adjustable magnitude in this construction is Z_{01}.

10.4. Electrical Compensation of Deviation in a Goniometric Radio Direction Finder

Compensation of Quadrant Deviation D sin 2q

Earlier it was shown that if fields created by field coils of the goniometer are not identical and are equal to

$$H_{1\,\text{макс}}\ \text{and}\ H_{2\,\text{макс}}=aH_{1\,\text{макс}},$$

where a ≠ 1, then in the direction finder there appears error, which is expressed (4.16):

$$\operatorname{tg}\Delta=\frac{\frac{a-1}{a+1}\sin 2\theta}{1-\frac{a-1}{a+1}\cos 2\theta}.$$

The dependence (4.16) of error on the arrival direction of the wave is analogous to quadrant deviation from the hull of a ship (5.57). This view can be used for compensation of quadrant deviation in goniometric radio direction finders, creating in the goniometer an error the opposite of the deviation. Field coils of the goniometer are made identical; to produce of unequal fluxes one of the field coils (longitudinal) is shunted by an inductance of a combine inductance and capacitance.

Let us consider first shunting of the field coil of the longitudinal loop by only an inductance, in which there is created an error opposite to quadrant deviation D sin 2q, and constant in the wave range.

Let us designate:

L_{long} — inductance of longitudinal loop;

L_{trans} — inductance of transverse loop;

L_1 — inductance of each field coil (we assume they are identical);

L_2 — shunting inductance;

h_e — virtual height of each loop.

Then the maximum flux in the goniometer from the transverse loop is equal to

$$H_{1\,\text{макс}} = \frac{kEh_e}{\omega(L_{\text{п}} + L_1)};$$

maximum flux from the longitudinal loop is equal to

$$H_{2\,\text{макс}} = \frac{kEh_e}{\omega\left(L_{\text{пр}} + \frac{L_1L_2}{L_1+L_2}\right)} \frac{L_2}{L_1+L_2} = aH_{1\,\text{макс}},$$

whence

$$a = \frac{1+\frac{L_1}{L_{\text{п}}}}{\frac{L_{\text{пр}}L_1}{L_{\text{п}}L_2} + \frac{L_{\text{пр}}}{L_{\text{п}}} + \frac{L_1}{L_{\text{п}}}}. \tag{10.1}$$

If $L_{trans} = L_{long} = L$, then

$$a = \frac{1+\frac{L_1}{L}}{1+\frac{L_1}{L_2}+\frac{L_1}{L}}. \tag{10.1'}$$

Knowing quadrant deviation of the radio direction finder, it is possible to select a in such a manner that

$$a = \frac{1-D}{1+D}, \tag{10.2}$$

i.e., so that there is created a deviation, opposite in sign to deviation from the hull. Then quadrant deviation of radio direction finder will be cancelled:

$$D = \frac{LL_1}{LL_1 + 2(LL_2 + L_1L_2)}. \tag{10.3}$$

From formula (10.1) the expression for shunting inductance L_2 will be

$$L_2 = \frac{aL_1L_{\text{пр}}}{L_{\text{п}} + L_1(1-a) - aL_{\text{пр}}}. \tag{10.4}$$

Usually the coil for shunting longitudinal loop (compensating choke) is made with leads, where they are chosen in such a manner as to compensate the coefficient of quadrant deviation D approximately each degree. Then after taking the deviation

curve it is simple to connect the corresponding lead of the coil.

Since from equations (10.3) and (10.4) it is clear that L_2 does not depend on wavelength, then the compensated quadrant deviation D will not depend on the wave.

Quadrant deviation, not depending on wavelength, is observed with waves, larger than 5-10 lengths of the ship hull.

In this calculation we disregarded capacitance of feeders, connecting the loops to field coils of the goniometer. If we consider these capacitances, then it turns out that the compensated coefficient of quadrant deviation D does not remain constant on different waves, but grows with decrease of wavelength. Sharpness of change of the compensated coefficient with the wave depends on the magnitude of the capacitance of the feeders. In practice the coefficient of quadrant deviation of a ship (aircraft) radio direction finder also grows with decrease of the wavelength. On long waves the capacitance of feeders is small, and the compensating choke can be calculated by formula (10.4).

Sometimes after connection of the compensational choke designed for the longest wave and determination on several waves of residual deviation it is found that on shorter waves there is a coefficient of quadrant deviation, increasing with decrease of wavelength. The means that abruptness of change of the compensated coefficient D with the wave is less than abruptness of change with the wave of real coefficient D. In such a case, in order to achieve the best compensation of quadrant deviation in the wave range, it is necessary to couple an additional capacitance in the transverse loop. Conversely, if steepness of variation of the compensated coefficient D with the wave is greater than steepness of variation of the real coefficient D, then one should couple the capacitance in the longitudinal loop [10.2].

In a two-path automatic radio direction finder it is possible to compensate the coefficient of quadrant deviation D, constant in the frequency range, by employing unequal gain factors in the channels.

Compensation of Quadrant Deviation E cos 2 q

During the analysis of the goniometric system we saw that presence of coupling between field coils of the goniometer leads to the appearance of quadrant error of form E cos 2q (§ 4.6), where (4.27)

$$E=\left(-\frac{Z_0}{Z}\right) \text{ radians,}$$

where Z_c — coupling impedance between field coils;

Z — impedance of loop circuit;

$\frac{Z_c}{Z}$ — has totally real value, if we disregard active components of Z_c and Z. Simultaneously there appears octant error K sin 4q, where

$$K = -\frac{1}{2}\left(\frac{Z_c}{Z}\right)^2 \text{ radians.}$$

By choice of coupling impedance Z_c it is possible to compensate quadrant deviation **E** cos 2q.

The simplest method of creating of coupling between field coils is connecting inductances (compensational chokes) between the field coils (Fig. 10.5).

In Fig. 10.5a we designate:

L — inductance of loop;

L_1 — inductance of field coil;

L_2 — inductance of coupling coil (compensational choke);

L_3 — inductance of searcher coil.

Let us disregard active resistances of loop circuits. We determine the expressions for Z and Z_c. We designate (Fig. 10.5b) resistance of the part of the circuit below points aa, but above points bb by X. Impedance of the loop circuit is

$$Z = j(\omega L + X).$$

We find the expression for Z_c from the influence of circuit II on circuit I:

$$Z_c = j(\omega L + X)\frac{LL_1}{LL_1 + 2(LL_2 + L_1L_3)}.$$

Compensated coefficient of deviation will be

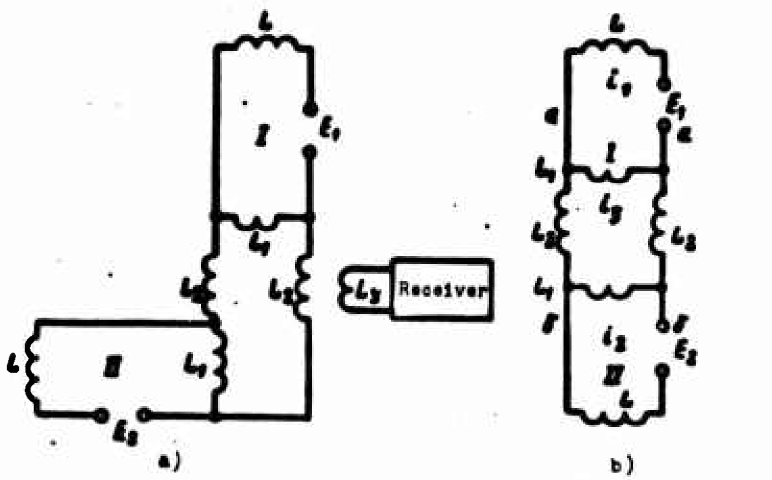

Fig. 10.5. Compensation of deviation **E** cos 2q.

$$\mathbf{E} = \frac{Z_c}{Z} = \frac{Z_c}{j(\omega L + X)} = \frac{LL_1}{LL_1 + 2(LL_2 + L_1L_3)}. \qquad (10.5)$$

Formulas (10.4) and (10.5) for compensated coefficients of deviation **E** and **D** are absolutely identical.

Consequently, inductance of coils L_2 for compensation of deviation E cos 2q should be calculated just as inductance of the coil for compensation of deviation D sin 2q.

A ship radio direction finder with a goniometer system usually is equipped with three identical compensational chokes with leads for compensation of deviation every 1°. One choke serves to compensate coefficient of deviation D, and we connect it in parallel to the longitudinal loop of the radio direction finder. Two other chokes serve to compensate coefficient of deviation E; they are connected between terminals of the field coils.

In comparison of mechanical and electrical methods of compensation of deviation one should note the basic deficiency of mechanical systems — lowering of accuracy of reading of bearing.

Let us assume that the design and electrical circuit of a radio direction finder permit with a given certain field strength of the fixed radio station, wavelength and average observer a certain accuracy of bearing reading (error Δq).

By a mechanical deviation compensator we compensate quadrant deviation

$$f = D\sin 2q.$$

The true bearing, corrected by the mechanical compensator, is

$$p = q + f = q + D\sin 2q.$$

Error of reading Δp will be obtained by differentiation of p with respect to q:

$$\Delta p = \Delta q(1 + 2D\cos 2q).$$

When q = 45, 135, 225 and 315°, $\Delta p = \Delta q$, i.e., accuracy of reading does not change.

When q = 0, 90, 180 and 270°, $\Delta p = \Delta q(1 \pm 2D)$, where D is expressed in radians.

Accuracy of the taken bearing depends on D. Thus, if $D = 15^{\circ}$; $2D = 30^{\circ} = \frac{\pi}{6} =$ $= 0.503$ rdn, then $\Delta p = 1.5\Delta q$.

With growth of D inaccuracy of reading of bearing on a radio direction finder with a mechanical compensator of deviation increases, so that work will be especially bad when D is great.

Electrical methods of compensation do not possess this deficiency.

The mechanical compensator of deviation creates additional error when there is an angle of silence (oscillation of the pointer of the indicator because of noises, etc.). Indeed let us assume that on the dial of the radio direction finder limits of the angle of silence will be

$$p_1 = q_1 + D\sin 2q_1,$$
$$p_2 = q_2 + D\sin 2q_2.$$

True bearing is equal to

$$p = \frac{q_1 + q_2}{2} + D\sin(q_1 + q_2).$$

Bearing on dial p_0 is defined as the arithmetic mean of p_1 and p_2:

$$p_0 = \frac{p_1 + p_2}{2} = \frac{q_1 + q_2}{2} + \frac{D}{2}(\sin 2q_1 + \sin 2q_2)$$

or

$$p_0 = \frac{q_1 + q_2}{2} + D\sin(q_1 + q_2)\cos(q_1 - q_2).$$

Error in reading of bearing will be determined from expression

$$|\Delta| = |p_0 - p| = D\sin(q_1 + q_2)[1 - \cos(q_1 - q_2)].$$

For any angle of silence, determined by $q_1 - q_2$, maximum error will be at $q_1 + q_3 =$ $= 90^\circ$. Then

$$|\Delta_{max}| = D[1 - \cos(q_1 - q_2)] = 2D\sin^2\left(\frac{q_1 - q_2}{2}\right).$$

If $D = 20^\circ$, $q_1 = 65^\circ$, $q_2 = 25^\circ$, i.e., angle of silence is 40°, then

$$|\Delta_{max}| = 40^\circ \sin^2 20^\circ = 4^\circ, 7.$$

10.5. Land (Airport, Shore) Radio Direction Finder

The antenna system of a ground radio direction finder should be installed as far as possible from re-emitting, current-conducting objects on a site with good ground conductivity. At the place of installation of a radio direction finder there should be no industrial interferences, impairing its sensitivity and accuracy. The place should be selected so that it is convenient as far as access roads, power supply, communication circuits, and so forth. It should be suitable for work in the RDF network (see Chapter 11).

Let us discuss in greater detail the fitness of the place from the point of view of ground conductivity and surroundings.

Requirements for ground conductivity are determined by the type of antenna system of the radiodirection finder. This question is considered in Chapter 6. If there is installed an antenna system, whose feeders must be protected from reception of a horizontally polarized electrical field, for instance a U-shaped goniometric or phase system of spaced vertical antennas, the ground conductivity on the antenna site should be at least $10^{-2}\ \frac{1}{\text{ohm}\cdot\text{m}}$ if the feeders are buried in the ground, and at least $10^{-3}\ \frac{1}{\text{ohm}\cdot\text{m}}$ if there is applied a ground metallizing network.

Ground conductivity can vary from month to month. For determination of average or minimum ground conductivity by separate measurements see [10.8]. Ground conductivity should be measured not only on the surface, but at the depth of laying

of feeders (about 2 m with burying of feeders). During measurements it is necessary to consider the depth of penetration of electromagnetic waves in the soil. In addition to Fig. 6.5 in Table 10.2 there are given for soils two values of depth conductivity where field strength decreases to 1/10 with respect to the field at the surface. One should also turn attention to whether ground conductivity is identical within the antenna site (see § 5.2).

In Table 10.3 there are given permissible minimum distances to certain reverse emitters or permissible angles of visibility when using an antenna system with a cosine directivity pattern [10.6]. With increase of separation of antennas errors from the influence of reverse emitters decrease (§ 5.3) and, correspondingly, distance to emitters can be decreased. Distances of Table 10.3 are given separately for long and medium waves (range of frequencies 100 Kc to 1.5 Mc), short waves (range of frequencies 1.5 to 30 Mc) and ultrashort waves (30 to 400 Mc) and for two cases:

Table 10.2. Depth of Penetration of Radio Waves, at Which Field Strength Decreases to 1/10.

Frequency, Mc	Depth of penetration, m	
	Bad conductivity $10^{-3}\ \frac{1}{\text{ohm}\cdot\text{m}} = 10^7$ CGSE	Normal conductivity $10^{-2}\ \frac{1}{\text{ohm}\cdot\text{m}} = 10^8$ CGSE
0.1	48	15
0.3	29	9
0.5	23	7
1.5	15	4
10	11	2.1
30	11	1.8
300	11	1.8

a) when expected mean quadratic error σ from the influence of the reverse emitter σ has a value on long, medium and short waves of 1°, and in UHF range 0.5°;

b) when expected mean quadratic error σ from influence of reverse emitter has value on long, average and short waves of 5°, and in UHF range 2°. The easier requirements pertain to the case when it is possible to permit the worst accuracy in light of other advantages of the place.

Simultaneously with errors there can be observed blurred minima during sound direction finding and an elliptical image of bearing in a two-channel visual radio direction finder, if phases of fields of the forward and reverse emitters do not coincide. In composing Table 10.3 it was assumed that the re-emitter creates maximum possible error without impairing the actual reading.

The table was composed on the basis of experimental materials and calculating formulas of errors from reverse emitters, having regular geometric forms (sphere, hemisphere, cube, mirror surface, vertical wire, horizontal wire, etc.).

Table 10.3. Requirement to Ensure Normal Work of a Radio Direction Finder

Number of point	Causes of error and parameters limited by them	Admissible values: Average, medium waves (100 Kc to 1.5 Mc) $\sigma = 1^\circ$	Average, medium waves $\sigma = 5^\circ$	Short waves (1.5 to 30 Mc) $\sigma = 1^\circ$	Short waves $\sigma = 5^\circ$	Ultrashort waves (30 to 400 Mc) $\sigma = 0.5^\circ$	Ultrashort waves $\sigma = 2^\circ$
I	Slope of section, deg	0.5	2	0.5	2	0.5	1
II	Vertical conductors — for a grounded one of length l = = 0.1 λ (vertical angle, deg) — for grounded one l = = 0.25 λ — for un-grounded one l = = 0.5 λ — for un-grounded one length several λ (distance)	10 $7l$	 l	 $7l$ 15 m	 $2l$	 $15l$ 30 m	 $4l$
III	Slanted conductors — in general — angle of inclination 45° (distance)	Approximately as in point II Half as large as in point II					
IV	Horizontal wires — telegraph at height h = 6–7 m — higher suspension H (distance, m)	180–200 (180–200)$\frac{H}{h}$	45 $45\frac{H}{h}$	See medium wavelengths			90
V	Wire barrier (distance, m)	Does not influence if grounded		90	45	180, 30 [10.7]	90
VI	Large conducting objects —square in area λ × λ — other area (distance, λ)	See point IX Varies in proportion to area of object		25	5	50	15

(Table 10.3 continued)

Number of point	Causes of error and parameters limited by them	Admissible values					
		Average, medium waves (100 Kc to 1.5 Mc)		Short waves (1.5 to 30 Mc)		Ultrashort waves (30 to 400 Mc)	
		$\sigma = 1^\circ$	$\sigma = 5^\circ$	$\sigma = 1^\circ$	$\sigma = 5^\circ$	$\sigma = 0.5^\circ$	$\sigma = 2^\circ$
VII	Forest —single trees —large group of trees (distance, m)	45 350	20 90	90 700	20 90	90 360	45 180
VIII	Isolated hills (vertical angle, deg)	2	5	2	5	Should not black out	
	Mountainous site	Not fit for installation of RDF					
IX	Buildings —small nonconducting (vertical angle, distance) —large conducting ones (distance, m)	2°	90 m 400	800 m			
X	Buildings housing RDF	If antennas are mounted in building it should be wooden					
XI	Takeoff and landing reinforced concrete airport runways (distance)	45 m	No influence	λ	No influence	λ	No influence
XII	River (distance, m)	No influence		No influence		300	No influence
XIII	Ditches, embankments (distance, m)	30	30	30	30	90	30
XIV	Railroad rails (distance, m)		90		90		

If the distance to the reverse emitter differs from that shown in the table, one should consider that field strength from the reverse emitter, and consequently also the error vary on long, medium and short waves: if the distance to the emitter is less than 0.1 λ it is inversely proportional to cube of distance D (by dependence $1/D^3$); if distance is more than λ, it varies as 1/D; in range of frequencies above 30 Mc with distance of several wavelengths, it varies as $1/D^2$.

Total mean quadratic error from several reverse emitters, each of which causes error σ_1, σ_2, σ_3, ..., is most correctly calculated by the formula

$$\sqrt{\sigma_1^2+\sigma_{11}^2+\sigma_3^2+\cdots}$$

The site chosen for installation of the antenna system after external inspection at a radius of several wavelengths usually is inspected for faults by a portable radio direction finder and local transmitter, carried or transported on motor vehicle, ship, or aircraft around the antenna system, or by direction finding of radio stations whose locations are known. The site is considered suitable for a radio direction finder, if mean quadratic error does exceed tolerable limits. So that it is possible to use a local transmitter, around its antenna at a radius of about 0.5 λ there should not be reverse emitters.

Distance from the transmitter to the center of the antenna system of the radio direction finder should be at least $(1-1.5)\lambda_{max}$ and 3-4 times greater than the separation of antennas in the antenna system of the radio direction finder.

Inspection of the site permits determining the order of errors expected during operation.

When constructing the home for a direction finder one should avoid construction of metal drain pipes and lightning rods. At a distance of about 150 m from the house electrical and telephone wiring must go into underground cable.

If the antenna system of vertical antennas is mounted around the location with the receiver-indicator, then power and communication cables should be brought in symmetrically to a pair of adjacent feeders, on the bisector of the angle between them.

Wiring inside the building should be very close to the floor, avoiding creation of loops (frames). If the radio direction finder is installed together with the transmitter, the antenna of the latter should be mounted symmetrically to the outdoor array of the direction finder, whose vertical antennas are set around the building. In the case of installation of several outdoor arrays on one site to solve the problem of the permissible minimum distance between antenna systems one should be guided by the fact that zenith angle at which one can see outdoor display from the center of the other, should be no more than 2-3°.

With a slanted site with slope permissible according to Table 10.3 vertical antennas in the system should be set normally to the plane of the site.

In a hilly site the best place is summit of a secluded round hill, which dominates the remaining hills.

The installed radio direction finder before use is calibrated by a local transmitter, observing here the shown requisite distances between transmitter and antenna system and satisfying requirements on surroundings of the transmitter antenna, or by distant radio stations.

The radio direction finder is calibrated also periodically during operation to check its accuracy. To permit periodic calibration dimensions of the direction finder site which are free from re-emitters are increased, since aound the antenna of the transmitter there also should be no reverse emitters. As a result of analysis of calibration materials we obrain an estimator of the accuracy of direction finding. If the obtained constant errors are confirmed by operation specifications of the radio direction finder, they are considered corrections to bearings. Sometimes results of calibration help us to find and remove the cause of large errors.

It is necessary to consider that errors from a local transmitter can differ from errors from distant radio stations because of noncoincidences of relative phases of fields of forward and reverse radiation when fixing local and distant transmitters.

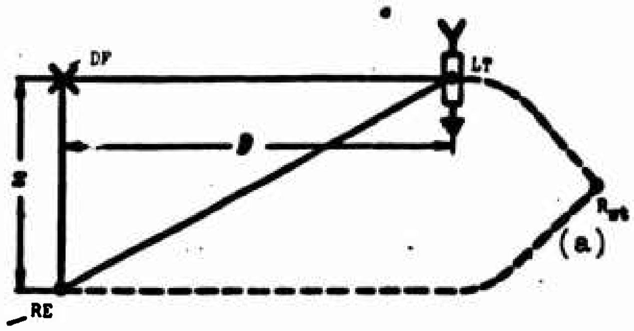

Fig. 10.6. Direction finding of near and distant transmitters.
KEY: (a) [Designation uncertain, R_{st} could be radio station].

Let us determine such a minimum distance to a local transmitter that errors from local and distant transmitters differ by not more than 10% (Fig. 10.6).

In the figure we designated:

DF — radio direction finder;

RE — reverse emitter at distance X from direction finder;

LT — local transmitter at distance D from direction finder (D >> X).

Difference of phases of fields of forward and reverse radiator during direction finding of a distant radio station will be

$$\varphi_1 = \frac{2\pi X}{\lambda}.$$

The same difference of phases from a local transmitter is

$$\varphi_2 = \varphi_1 + \Delta\varphi = \frac{2\pi}{\lambda}[X + (\sqrt{D^2 + X^2} - D)] \approx \frac{2\pi}{\lambda}\left(X + \frac{X^2}{2D}\right)$$

or

$$\Delta\varphi \approx \frac{\pi X^2}{\lambda D}.$$

Error in direction finding is proportional to the cosine of the difference of phases of the fields of the forward and reverse radiators.

To satisfy the formulated requirement for permissible difference of errors during direction finding of local and distant transmitters, condition $\frac{\cos \varphi_2}{\cos \varphi_1} \geq 0.9$ should be satisfied. Since, for $\frac{\pi}{2} > \varphi_1 > 0$ $\cos \varphi_2 = \cos(\varphi_1 + \Delta\varphi) \approx \cos \varphi_1 - \Delta\varphi \sin \varphi_1$, $\frac{\cos \varphi_2}{\cos \varphi_1} = 1 - \Delta\varphi \tan \varphi_1$ and $\Delta\varphi \tan \varphi_1 \leq 0.1$ or we should have

$$D > \frac{10 a X^2}{\lambda} \operatorname{tg} \varphi_1.$$

Consequently, when X = 100 m we should have conditions:

for $\lambda = 300$ m, $D \geq 1.7$ km; for $\lambda = 12$ m, $D \geq 45$ km.

On short waves these requirements are not practically feasible. Even if calibration is performed at the required distance, error varies with the frequency, azimuth and height of location of the transmitter. In Table 10.4 there are calculated changes of frequency, azimuth and angle of inclination of the wave front, corresponding to change of error of bearing from the influence of the reverse radiator from a maximum value down to zero. It is assumed that the reverse radiator is a distance of 100 m from the antenna system of the radio direction finder.

Table 10.4. Changes of Frequency, Azimuth and Angle of Inclination of the Wave Front, Corresponding to Change of Error from a Maximum to Zero

Frequency, Mc	1 Mc	25 Mc
Change of frequency, %	37	1.48
Change of azimuth, deg	44	1.75
Change of angle of inclination of wave front, deg	75	15

From the presented it follows that calibration by a local transmitter cannot reveal systematic errors of the radio direction finder, which are found by selection of bearings by distant radio stations, whose locations are known, and as a result of prolonged study of the operation of the radio direction finder.

MT-65-58
Principles of Radio Direction Finding,
"Soviet Radio" Publishing House,
Moscow, 1964.
Pages: 571-597

C H A P T E R 11

ACCURACY OF POSITION FINDING BY RADIO BEARINGS

List of Designations Appearing in Cyrillic

к = circ = circular

мин = min = minimum

э = ell = ellipse
э = op = operational

П = RDF = radio direction finder

In order to obtain good results in direction finding, it is necessary to have radio direction finders, possessing sufficient accuracy and sensitivity, which should be properly situated and correctly used. It is necessary also to be able for each fix to estimate bearings and to find from bearings and estimations of them the most likely place or region for finding the fixed target.

In the present chapter we consider methods of appraisal of a single bearing and accuracy of position finding from bearings of n radio direction finders, and also methods of construction of working zones of two radio direction finders.

At the base of chapter lies application of the statistical theory of errors to radio direction finding.

11.1. Methods of Estimating a Single Bearing

Errors during direction finding, as shown in § 2.4, are divided in errors of the system, which can be allowed for in the form of corrections, and random, which cannot be allowed for by corrections. Random errors characterize an individual reading of bearing.

In order to determine the position of a target and to calculate position error from bearings of n radio direction finders, it is necessary to know estimator of

errors of bearings. On the basis of large number of observations, conducted over a prolonged time, there can be found the mean quadratic operational erro radio direction finder σ_{op}. However, estimated mean quadratic error of dividual bearing may differ considerably from σ_{op}, especially on short waves.

Sometimes we use subjective estimation of the bearing by the operator, for which we beforehand establish 4-5 categories of bearings (for instance, by quality of readings, by degree of their stability, etc). For each category of bearing we experimentally establish the mean quadratic angular error. Subjective estimation with known accuracy can be applied only on ultrashort, medium, and long waves, since on these wave ranges mean quadratic error for the outlined categories of bearings should be preserved. On shortwaves, where there are too many factors affecting the accuracy of direction finding, subjective estimation may lead to incorrect results.

It is better to employ objective estimation of bearings [11.4, 11.5], which is based on the physical picture of propagation and conditions of direction finding of short waves. We give in [11.5] a method of estimating bearings on short waves. Analysis of random errors of a short-wave radio direction finder shows that they can be divided into three statistically non-connected groups with dispersions V_1, V_2, V_3, deg^2.

1. Errors varying very slowly in time – error of instruments and from the influence of the position (near and distant surroundings). For the dispersion of these errors it is possible to write:

$$V_1 = V_{11} + V_{12}(f),$$

where V_{11} – a component which does not depend on frequency. V_{11} varies from 0.1 deg^2 to 1 deg^2 depending upon the antenna system of the radio direction finder and the site of its installation; $V_{12}(f)$ – component, depending on frequency. On the basis of operational specifications one may assume that $V_{12}(f) = Ab$, where A – constant coefficient, depending on the quality of the installation site and separation of the antenna system; b – coefficient, dependent on frequency:

- for frequencies 2-4 Mc, b ≈ 1;
- for frequencies 4-9 Mc, b ≈ 2;
- for frequencies above 9 Mc, b ≈ 3.

2. Errors varying slowly in time – errors from lateral deviation of radio waves during reflection from the ionosphere. This error depends on the distance and, to an extent, on the time of day; because of the large period of change this error usually is not averaged during the time of taking a bearing (§ 6.4). Values

of dispersion of lateral deviation V_2 deg^2 = $\Phi(D)$ are given in Fig. 6.18 for a small-base antenna system.

3. Errors varying rapidly in time – errors from interference of radio waves and polarization; here there enters subjective error of operator readings. These errors are averaged by the operator. The operator usually takes several (5-12) averaged readings, each of which is the result of observations for 5-10 sec. With greater duration of each separate averaged reading the operator, who, as a rule, recalls the picture of the bearings only for the last 5-10 sec, will waste part of the time of direction finding.

Let us assume that the operator took n averaged readings, obtained average bearing θ, the difference between maximum and minimum values of averaged reading r deg, and observed during the time of taking of a separate averaged reading variation of the bearing δ deg.

Dispersion V_3 is calculated by the formula

$$V_3 = V_{31} + V_{32}.$$

Proceeding from the normal law of distribution of errors of bearings

$$V_{31} \approx \frac{r^2}{n}, \ \text{deg}^2.$$

Component of dispersion V_{32} considers subjective error of the operator. It depends on limits of variation of bearing δ during the time of taking averaged readings on a cathode-ray tube or on the angle of silence during a sound method of direction finding by a minimum.

In [11.5] it is proposed to determine V_{32} on the basis of magnitude of play of the bearing during the time of an averaged reading or the angle of silence within limits:

$$\begin{aligned}
0\text{–}8^\circ &\quad V_{32} = 0 \ \text{deg}^2;\\
9\text{–}13^\circ &\quad V_{32} = 1 \ \text{deg}^2;\\
14\text{–}18^\circ &\quad V_{32} = 2 \ \text{deg}^2;\\
19\text{–}23^\circ &\quad V_{32} = 4 \ \text{deg}^2;\\
24\text{–}37^\circ &\quad V_{32} = 6 \ \text{deg}^2;\\
\text{to above } 38^\circ &\quad V_{32} = 9 \ \text{deg}^2;
\end{aligned}$$

Thus, total dispersion of error of bearing will be

$$V = V_1 + V_2 + V_3,$$

where

$$V_1 = V_{11} + V_{12}(f),\ V_2 = \Phi(D),\ V_3 = V_{31} + V_{32}.$$

Dispersion V, or mean quadratic error $\sigma = \sqrt{V}$, together with mean bearing θ are used for plotting.

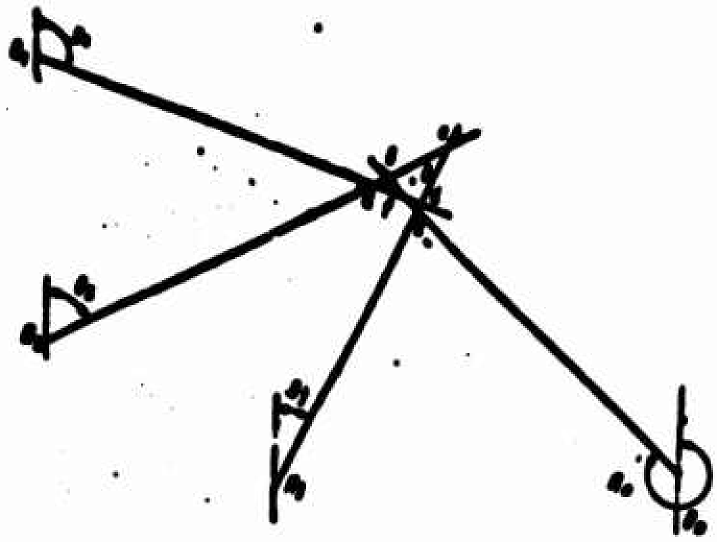

Fig. 11.1. Plotting bearings.

The indicated coefficients for calculation of components of dispersion are empirical and must be definitized on the basis of a check direction finding of radio stations, whose positions are known.

11.2. Ellipse of Error with n Radio Direction Finders

Let us assume that a radio station, located at point O (Fig. 11.1) is fixed by n radio direction finders; $(RDF)_1$, $(RDF)_2$,..., $(RDF)_n$, and there are obtained bearings θ_1, θ_2, ..., θ_n. From angular errors of bearings as a result of plotting we formed a polygon of fixing (abcdef).*

We designate:

Δ_1, Δ_2, ..., Δ_n – angular errors of radio bearings;

σ_1, σ_2, ..., σ_n – mean quadratic angular errors of bearings.

In Fig. 11.2 there is depicted line of bearing of one (j-th) radio direction finder. Let us place the origin of coordinates at point O, the true position of the radio station, axis OX we direct along parallel, axis OY along the meridian of point O. Let us designate by p_j the length of the perpendicular from point O to the line of bearing $(RDF)_jK$: $p_j = D_j\Delta_j$, where D_j – distance between $(RDF)_j$ and O.

Let us assume that as a result of plotting lines of bearings on a map for the position of the radio station we take point S with coordinates x, y, at distance q_j from line of bearing $(RDF)_jK$.

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*We consider that bearings can be plotted in the form of straight lines.

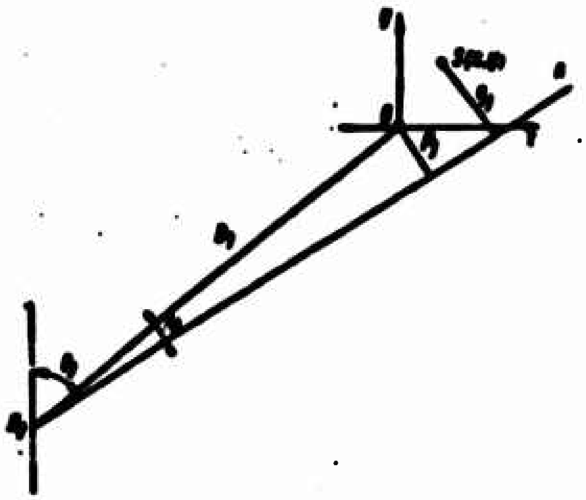

Fig. 11.2. Calculating probability density.

For q_j it is possible to write:

$$q_j = p_j + x\cos\theta_j - y\sin\theta_j.$$

The probability that the distance from point S to line of bearing $(RDF)_j K$ lies between q_j and $q_j + dq_j$, is determined by formula

$$P(q_j)\,dq_j = \frac{1}{\sqrt{2\pi}\,\sigma_j D_j}\, e^{-\frac{q_j^2}{2\sigma_j^2 D_j^2}}\, dq_j,$$

where $\sigma_j D_j = E_j$ – mean quadratic deviation of the line of bearing from the true position of the radio station.

Analogous expression can be written for other lines of bearings. Total probability that the assumed location of the radio station S is found from n lines of bearings, taken from points $(RDF)_1$, $(RDF)_2$, ..., $(RDF)_n$, at distance from q_1 to $q_1 + dq_1$, from q_2 to $q_2 + dq_2$, ..., from q_n to $q_n + dq_n$, correspondingly, will be

$$P(q_1, q_2, \ldots, q_n)\,dq_1 dq_2 \ldots dq_n = \frac{1}{(2\pi)^{\frac{n}{2}} E_1 E_2 \ldots E_n}\, e^{-\frac{1}{2}\sum_{j=1}^{n}\frac{(p_j + x\cos\theta_j - y\sin\theta_j)^2}{E_j^2}}\, dq_1 dq_2 \ldots dq_n. \quad (11.1)$$

From the principle of least squares coordinates (x_0, y_0) of the most probable position of the fixed radio station are found from the condition of a maximum of expression (11.1) or a minimum of exponent e. To determine x_0, y_0 one should equate to zero derivatives with respect to x and y of exponent e in (11.1). We obtain two equations:

$$\sum_{j=1}^{n}\frac{p_j\cos\theta_j}{E_j^2} + Ax - By = 0,$$

$$\sum_{j=1}^{n}\frac{p_j\sin\theta_j}{E_j^2} + Bx - Cy = 0,$$

from which it follows that

$$\left.\begin{aligned} x_0 &= \frac{1}{AC - B^2}\sum_{j=1}^{n}\left[p_j\,\frac{B\sin\theta_j - C\cos\theta_j}{E_j^2}\right],\\ y_0 &= \frac{1}{AC - B^2}\sum_{j=1}^{n}\left[p_j\,\frac{A\sin\theta_j - B\cos\theta_j}{E_j^2}\right]. \end{aligned}\right\} \quad (11.2)$$

where

$$\left.\begin{aligned}A&=\sum_{k=1}^{n}\frac{\cos^2\theta_k}{E_k^2};\\ B&=\sum_{k=1}^{n}\frac{\sin\theta_k\cos\theta_k}{E_k^2};\\ C&=\sum_{k=1}^{n}\frac{\sin^2\theta_k}{E_k^2}.\end{aligned}\right| \tag{11.3}$$

Point (x_0, y_0) the most likely place of finding the radio station and is called the center of probability.

Let us transfer the origin of coordinates to point (x_0, y_0). Then, the probability of finding the radio station at any point with coordinates (x, y) will be

$$P(x, y)dxdy=\frac{\sqrt{AC-B^2}}{2\pi}e^{-\frac{1}{2}(Ax^2-2Bxy+Cy^2)}dxdy. \tag{11.4}$$

From the right side of equality (11.4) we see that only exponent e depends on variables x and y. Consequently, probability P varies only with change of exponent e. Assuming that the exponent is equal to a constant, we obtain the equation of a contour, on whose boundaries the probability of finding the radio station within limits of elementary areas dxdy has a constant value, i.e., the equation of the contour of constant probability density. Let us designate exponent e by coefficient $-\frac{1}{2}K_0^2$. Then expression

$$Ax^2-2Bxy+Cy^2=K_0^2 \tag{11.5}$$

determines the locus of points with identical probability density $P(x, y)\,dxdy =$ $=$ const. Equation (11.5) is the equation of an ellipse with its center at point (x_0, y_0), i.e., contours of constant probability density are ellipses with their center of the center of probability.

To determine the integral probability P_{ell} of finding the object of direction finding inside the ellipse constructed for the given value of K_0, one should integrate the expression for differential probability (11.4) within the area of the ellipse S_{ell}:

$$P_e=\iint_S\left[\frac{\sqrt{AC-B^2}}{2\pi}e^{-\frac{1}{2}(Ax^2-2Bxy+Cy^2)}\right]dxdy.$$

We replace axes x, y by axes x', y', turning them angle γ, where $\tan 2\gamma = \frac{2B}{C - A}$.

Then we replace axes x', y' by axes x'', y'':

$$x'' = \frac{x'}{a_0}, \quad y'' = \frac{y'}{b_0},$$

where a_0 and b_0 — semiaxes of a unit ellipse.

$$a_0, b_0 = \frac{\sqrt{2}}{\sqrt{A + C \mp \sqrt{(A - C)^2 + 4B^2}}}.$$

Let us turn to polar coordinate axes

$$\rho^2 = x''^2 + y''^2$$

and

$$dx''dy'' = \rho d\rho d\varphi.$$

Making these replacements, we obtain the following expression for integral probability P_{ell}:

$$P_0 = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{K_0} e^{-\frac{1}{2}\rho^2} \rho d\rho.$$

We consider $\rho^2 = 2u$, then $\rho d\rho = du$:

$$P_0 = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{0.5K_0^2} e^{-u} du.$$

Integral probability is determined by formula

$$P_0 = 1 - e^{-0.5K_0^2},$$

from which

$$K_0 = \sqrt{-2\ln(1 - P_0)}. \qquad (11.6)$$

Thus, K_0 in equation (11.5) is calculated by formula (11.6), depending upon the given integral probability P_{ell}. Below we give coefficients K_0 for various values of P_{ell} (Fig. 11.3 and Table 11.1).

If $K_0^2 = 1$, we obtain the equation of the so-called unit ellipse $Ax^2 - 2Bxy + Cy^2 = 1$, and $P_{ell} = 39.4\%$.

We combine axes of the right-angle system of coordinates with principal axes

of the ellipse of probability. Equation (11.5) of the ellipse takes the form

$$\frac{x^2}{a_0^2}+\frac{y^2}{b_0^2}=K_0^2,$$

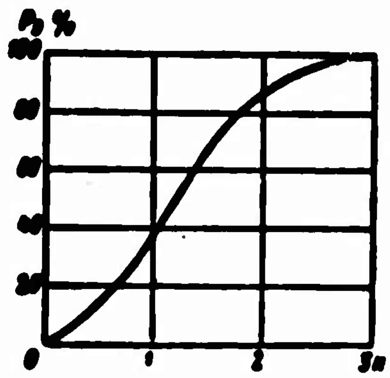

Fig. 11.3. Curve of integral probability.

Table 11.1

P_0	K_0	P_0	K_0	P_0	K_0
0	0	0,3	0,842	0,7	1,552
0,05	0,322	0,394	1	0,865	2
0,1	0,458	0,5	1,179	0,989	3
0,12	0,5	0,6	1,353		
0,2	0,671	0,632	1,41		

where a_0 and b_0 – semimajor and semiminor axes of any ellipse. On the basis of (11.5) expressions for semiaxes of the ellipse will be

$$a_0,\ b_0=\frac{\sqrt{2}K_0}{\sqrt{A+C\mp\sqrt{(A-C)^2+4B^2}}}. \qquad (11.7)$$

Upper sign (-) pertains to semiaxis a_0 (major), lower sign (+) pertains to semiaxis b_0 (minor).

We substitute in (11.7) formula (11.6), and after transformations we obtain

$$a_0,\ b_0=\sqrt{-\ln(1-P_0)}\,\frac{\sqrt{A+C\pm\sqrt{(A-C)^2+4B^2}}}{\sqrt{AC-B^2}}. \qquad (11.8)$$

Angle γ between the major axis of the ellipse $2a_0$ and the meridian is determined from (11.5) expression

$$\operatorname{tg}2\gamma=\frac{2B}{C-A} \qquad (11.9)$$

or

$$\operatorname{tg}\gamma=\frac{A-C-\sqrt{(A-C)^2+4B^2}}{2B}. \qquad (11.9')$$

For two radio direction finders formula (11.8) for calculation of semiaxes of the ellipse of probability is simplified and takes the form

$$a_0,\ b_0=\sqrt{-\ln(1-P_0)}\times$$
$$\times\frac{D_1\sigma_1 D_2\sigma_2}{\sqrt{D_1^2\sigma_1^2+D_2^2\sigma_2^2\mp\sqrt{(D_1^2\sigma_1^2+D_2^2\sigma_2^2)^2-4D_1^2\sigma_1^2D_2^2\sigma_2^2\sin^2\alpha_{12}}}}. \qquad (11.10)$$

where $\alpha_{12} = (\theta_2 - \theta_1)$ – angle of intersection of bearings.

For the angle of orientation of the major axis of the ellipse from (11.9) we obtain

$$\operatorname{tg} 2\gamma = \frac{D_1^2 e_1^2 \sin 2\theta_1 + D_2^2 e_2^2 \sin 2\theta_2}{D_1^2 e_1^2 \cos 2\theta_1 + D_2^2 e_2^2 \cos 2\theta_2}. \tag{11.11}$$

Knowing a_0 and b_0, we can find the area of the ellipse of probability:

$$S_0 = \pi a_0 b_0 = 2\pi \ln(1 - P_0) \frac{1}{\sqrt{AC - B^2}}. \tag{11.12}$$

For two intersecting bearings the area of the ellipse will be

$$S_0 = 2\pi \ln(1 - P_0) \frac{E_1 E_2}{\sin \alpha_{12}}. \tag{11.13}$$

Using formulas (11.8) and (11.9'), we can obtain the following expressions for semiaxes of the ellipse:

$$\left.\begin{aligned} a_0^2 &= \frac{K_0^2}{C + B \operatorname{tg} \gamma}, \\ b_0^2 &= \frac{K_0^2}{A - B \operatorname{tg} \gamma}. \end{aligned}\right\} \tag{11.14}$$

In [11.7] there is offered a graphical method of determination parameters of the ellipse of probability using a special plotting board and formulas (11.9) and (11.14) at distances, where bearings can be plotted in the form of straight lines (see Table 12.1).

The method is as follows.

Let us assume that as a result of plotting bearings we have found the center of probability O (Fig. 11.4a). Let us pass to O from the point of location of the radio direction finder $(RDF)_j$ line of bearing $(RDF)_j O$ at angle θ_j to meridian OM. We pass a second line $(RDF)_j K$ at angle σ_j to line $(RDF)_j O$. We place at O a transparent plotting board with rectangular axes, where axis OY is matched with the meridian at O. We mark the points a_j and b_j of the intersection of line $(RDF)_j K$ with axes OY of the plotting board.

From Fig. 11.4a it follows that

$$D_j e_j \approx OL, \quad Oa_j = \frac{D_j e_j}{\sin \theta_j} \quad \text{and} \quad Ob_j = \frac{D_j e_j}{\cos \theta_j}.$$

Axes OY and OX on the plotting board have scale with calibrations $\frac{1}{(OY)^2}$ and $\frac{1}{(OX)^2}$, correspondingly, where in the denominators are lengths in the scale of map being used.

Therefore, readings on axes of the plotting board will be

$$Oa_j = \frac{\sin^2\theta_j}{D_j^2 e_j^2},\ Ob_j = \frac{\cos^2\theta_j}{D_j^2 e_j^2} \text{ and } \sqrt{Oa_j Ob_j} = \frac{\sin\theta_j \cos\theta_j}{D_j^2 e_j^2}.$$

We add readings on axes of the plotting board for all radio direction finders. Then

$$\sum_{j=1}^{n} Oa_j = C,\ \sum_{j=1}^{n} Ob_j = A \text{ and } \sqrt{\sum_{j=1}^{n} Oa_j Ob_j} = B,$$

where A, B, and C correspond to formulas (11.3). Sign for B is shown on the plotting board.

Finding A, B, and C, it is possible to calculate a_0, b_0 and γ from formulas (11.14) and (11.9).

The plotting board is shown in Fig. 11.4b.

It is possible to characterize linear error by the mean quadratic value. In this case it is called by circular error of plotting (R). By definition

$$R = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \rho^2 d\varphi}, \qquad (11.15)$$

where ρ — radius, from center of ellipse of errors;

φ — angle of radius with initial line of reading, e.g., with the major axis of the ellipse.

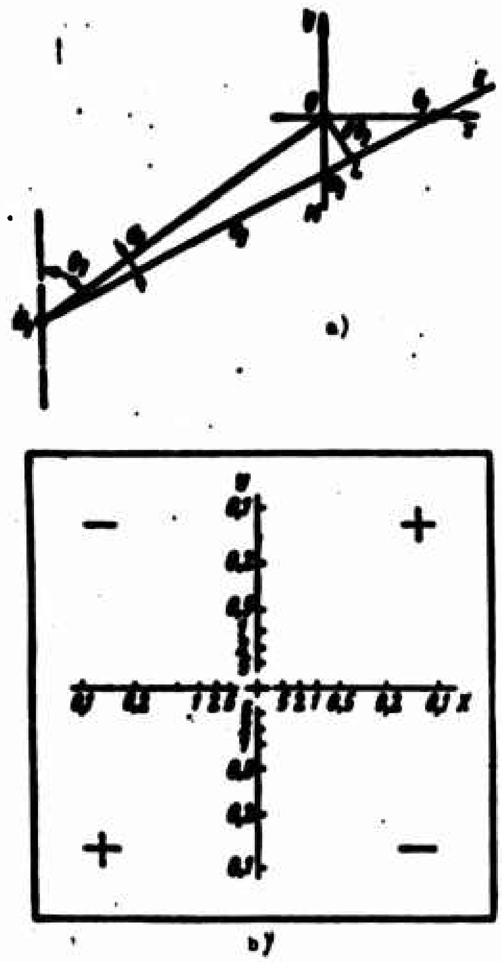

Fig. 11.4. Principle of use of a plotting board for determination parameters of the ellipse of probability: a) diagram of plotting on map; b) form of plotting board.

Let us rewrite (11.15) in the form

$$R = \sqrt{\frac{1}{2\pi}\left[2\int_0^{\pi/2}(\delta_1^2 + \delta_2^2)\,d\varphi\right]}, \qquad (11.15')$$

where δ_1 and δ_2 — conjugate radii of the ellipse of errors.

Let us replace in (11.15') [1.17]

$$\delta_1^2 + \delta_2^2 = a_0^2 + b_0^2$$

and substitute for a_0, b_0 expressions (11.8). Then

$$R = \sqrt{\frac{1}{2}(a_0^2 + b_0^2)} = \sqrt{-\ln(1 - P_0)}\sqrt{\frac{A + C}{AC - B^2}}. \qquad (11.16)$$

If $-\ln(1-P_{ell})=1$, circular error is

$$R_0=\sqrt{\frac{A+C}{AC-B^2}}. \tag{11.17}$$

Integral probability of mean quadratic linear error R_0 for $a_0 = b_0$ coincides with integral probability P_{ell} of elliptic error, and according to Fig. 11.3, $P_{ell} = 63.2\%$.

For other ratios $\frac{b_0}{a_0}$ integral probability of circular error R_0 increases somewhat, and as $\frac{b_0}{a_0}\to 0$ it is equal to 68%.

Below there are given values of integral probability of circular error R_{circ} (in %) for various $\frac{R}{R_0}$ for extreme values $\frac{b_0}{a_0}=1$ and $\frac{b_0}{a_0}=0$.

Table 11.2. Integral Probability R_{circ} in % Depending upon $\frac{R}{R_0}$ and $\frac{b_0}{a_0}$

	$\frac{R}{R_0}$	0,5	0,8	1	1,5	2	2,5	3
P_c	$\frac{b_0}{a_0}=0$	39	58	68	86	95	99	100
	$\frac{b_0}{a_0}=1$	20	48	63	90	99	100	

For two radio direction finders from general expression (11.17) for R_0 we have

$$R_0=0{,}01745\frac{\sqrt{\sigma_1^2D_1^2+\sigma_2^2D_2^2}}{\sin\alpha_{12}}, \tag{11.18}$$

where σ_1 and σ_2 are expressed in degrees.

Let us return to expressions (11.2) for coordinates of the center of probability; we shall find a method of calculating them. First we determine the point of intersection of bearings from any two radio direction finders, for instance, $(RDF)_j$ and $(RDF)_k$. Equations of lines of bearings we obtain from Fig. 11.2:

$$\left.\begin{aligned}\rho_j+x\cos\theta_j-y\sin\theta_j=0,\\ \rho_k+x\cos\theta_k-y\sin\theta_k=0.\end{aligned}\right\} \tag{11.19}$$

Coordinate of points of intersection of bearings from $(RDF)_j$ and $(RDF)_k$ from (11.19) will be:

$$\left.\begin{aligned}x_{jk}&=\frac{\rho_k\sin\theta_j-\rho_j\sin\theta_k}{\cos\theta_j\sin\theta_k-\sin\theta_j\cos\theta_k}=\\&=\frac{\rho_k\sin\theta_j-\rho_j\sin\theta_k}{\sin(\theta_k-\theta_j)},\\ y_{jk}&=\frac{\rho_k\cos\theta_j-\rho_j\cos\theta_k}{\sin(\theta_k-\theta_j)}.\end{aligned}\right\} \tag{11.20}$$

Let us introduce for characteristic of the weight of the point of intersection of the j-th and k-th bearings as an estimate of the weight of magnitude m_{jk}, the inverse of the square of the area of the unit ellipse of probability, obtained if the radio station is fixed by two radio direction finders, the j-th and k-th:

$$m_{jk} = \frac{\pi^2}{S^2_{ell\,jk}},$$

where $S_{ell\ jk}$ – area of unit ellipse of probability,

$$S_{ell\,jk} = \pi \frac{E_j E_k}{\sin \alpha_{kj}};$$

α_{kj} – angle of intersection of j-th and k-th bearings, equal to $\theta_k - \theta_j$;

$$m_{jk} = \frac{\sin^2(\theta_k - \theta_j)}{E_j^2 E_k^2}. \tag{11.21}$$

We obtain from formulas (11.20) and (11.21) products $x_{jk}m_{jk}$ and $y_{jk}m_{jk}$:

$$x_{jk}m_{jk} = \frac{1}{E_j^2 E_k^2}(p_k \sin\theta_j - p_j \sin\theta_k)\sin(\theta_k - \theta_j),$$

$$y_{jk}m_{jk} = \frac{1}{E_j^2 E_k^2}(p_k \cos\theta_j - p_j \cos\theta_k)\sin(\theta_k - \theta_j).$$

It is possible to show that

$$\left.\begin{aligned}
\sum_{j=1}^{n}\sum_{k=1}^{n} x_{jk}m_{jk} &= \sum_{j=1}^{n} p_j \frac{B\sin\theta_j - C\cos\theta_j}{E_j^2},\\
\sum_{j=1}^{n}\sum_{k=1}^{n} y_{jk}m_{jk} &= \sum_{j=1}^{n} p_j \frac{A\sin\theta_j - B\cos\theta_j}{E_j^2},\\
\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk} = \sum_{j=1}^{n}\sum_{k=1}^{n} &\frac{\sin^2(\theta_k - \theta_j)}{E_j^2 E_k^2} = AC - B^2,
\end{aligned}\right\} \tag{11.22}$$

where A, B, and C are determined earlier and are expressed by formulas (11.3). Comparing expression (11.2) and (11.22), we arrive at the conclusion that coordinates of the center of probability x_0, y_0 can be calculated by formulas

$$\left.\begin{aligned}
x_0 &= \frac{\sum_{j=1}^{n}\sum_{k=1}^{n} x_{jk}m_{jk}}{\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}},\\
y_0 &= \frac{\sum_{j=1}^{n}\sum_{k=1}^{n} y_{jk}m_{jk}}{\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}}.
\end{aligned}\right\} \tag{11.23}$$

Formulas (11.23) are analogous to formulas for calculating the center of gravity of masses m_{jk}, located at the intersection points of lines of bearings.

Thus, coordinates of the center of probability x_0, y_0 can be defined as coordinate of the center of gravity of the figure of fixing abcdef ... (Fig. 11.1), at vertices of which are placed masses m_{jk} (11.21), characterizing these vertices.

For two bearings the center of probability coincides with the point of intersection of bearings.

For three radio direction finders masses of vertices of the fixing triangle will be:

$$m_{12}=\frac{\sin^2\alpha_{12}}{E_1^2E_2^2},\quad m_{13}=\frac{\sin^2\alpha_{13}}{E_1^2E_3^2},\quad m_{23}=\frac{\sin^2\alpha_{23}}{E_2^2E_3^2};$$

or, multiplying numerators and denominators by identical factors, it is possible to write them otherwise:

$$m_{12}=\frac{E_3^2\sin^2\alpha_{12}}{E_1^2E_2^2E_3^2},\quad m_{13}=\frac{E_2^2\sin^2\alpha_{13}}{E_1^2E_2^2E_3^2},\quad m_{23}=\frac{E_1^2\sin^2\alpha_{23}}{E_1^2E_2^2E_3^2}. \tag{11.24}$$

Cancelling identical denominators in expressions (11.24), we obtain for masses

$$m_{12}=D_3^2\sigma_3^2\sin^2\alpha_{12},\quad m_{13}=D_2^2\sigma_2^2\sin^2\alpha_{13},\quad m_{23}=D_1^2\sigma_1^2\sin^2\alpha_{23}. \tag{11.25}$$

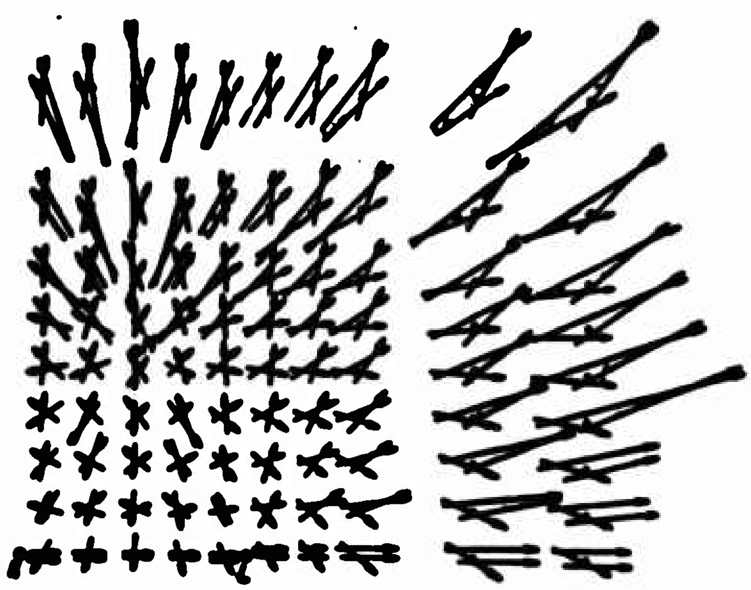

Fig. 11.5. Construction of centers of probability with three radio direction finders.

Graphic determination of the center of probability for three bearings consists in finding the point of intersection of straight lines, drawn from vertices of the fixing triangle so that opposite sides are divided by these lines into segments, inversely proportional to masses of the vertices, adjacent to sides.

In practice we usually mark the center of probability intuitively, proceeding from the position

that the sharper the angle of intersection of bearings at a vertex and the smaller the product $D\sigma$ for the bearing opposite the vertex, the more one should be removed from this vertex.

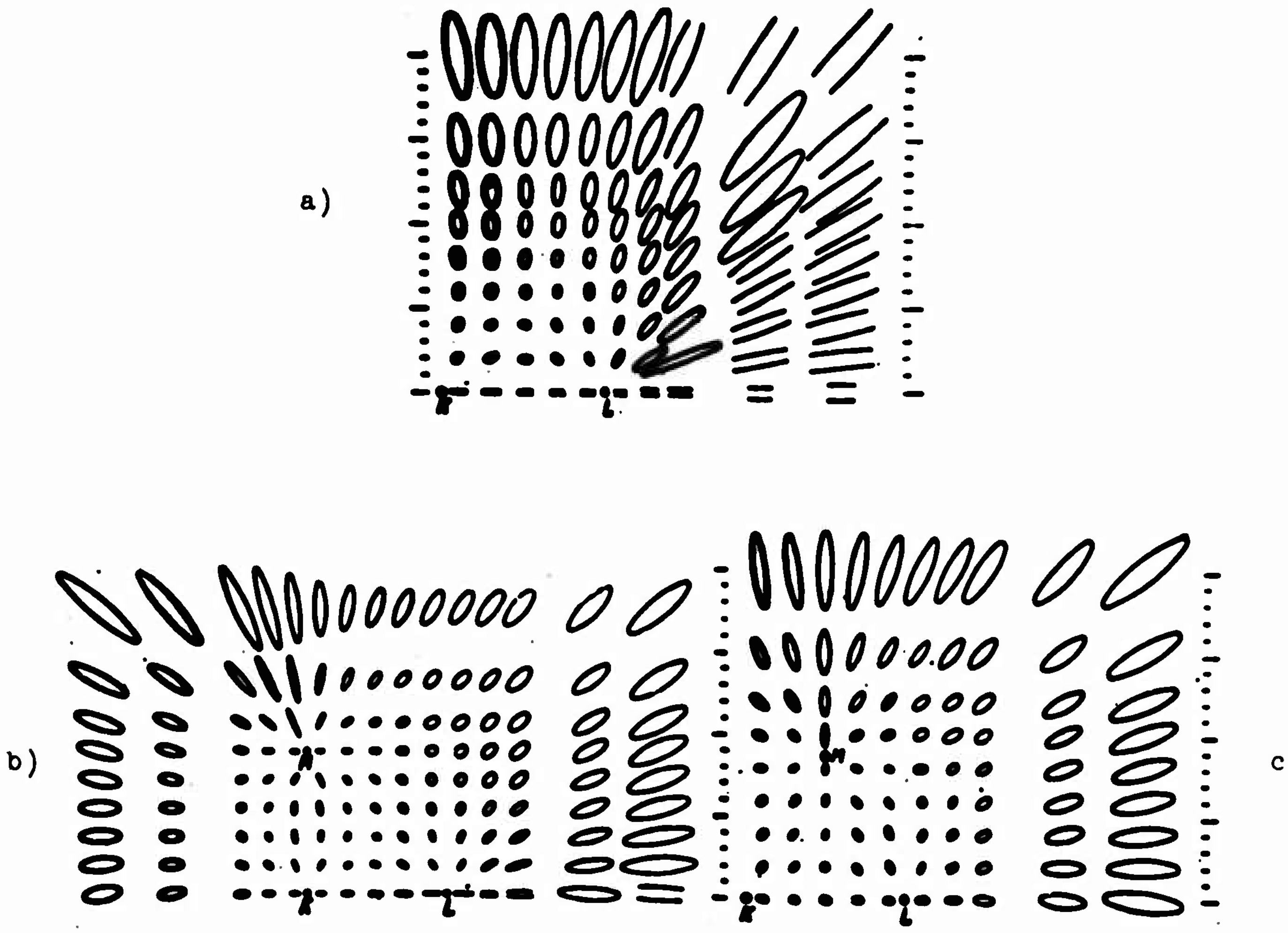

Fig. 11.6. Construction of ellipses of probability for the case of a mean quadratic error of 2° and $P_{ell} = 50\%$: a) two radio direction finders; b) three radio direction finders at vertices of right-angle triangle; c) three radio direction finder at vertices of an equilateral triangle.

If the center of probability is found and a_0, b_0 and γ, are calculated it is possible to construct the ellipse of probability graphically.

In Fig. 11.5 typical examples of construction of centers of probability are shown for the case of three radio direction finders, located at points K, L, M where $\sigma_1 = \sigma_2 = \sigma_3 = 2^{\circ}$.

In Fig. 11.6 are given constructed ellipses of probability for several cases of distribution of radio direction finders [11.6].

Calculations of the center of probability, dimensions of the ellipse of

probability, and orientation of the major axis of the ellipse are given for the case of flat ground, i.e., at distances from the radio direction finders up to 500-800 km. With greater distances it is necessary to allow for curvature of the earth. Equations of lines of bearings must be modified accordingly.

If the lines of bearings are plotted on a map taking into account curvature of the earth and we obtained a fixing figure, then the center of probability and parameters of the ellipse (or circle) of probability can be calculated by the presented theory or by the described graphical method.

11.3. <u>Region, Serviced by Two Radio Direction Finders</u>

Let us determine boundaries of a region, within which linear error of direction finding with required probability will not exceed a given value. The area inside the boundaries of this region is called the working zone of two radio direction finders.

For a given probability the maximum distance at which an object of direction finding can be located from its most probable position, is the major semiaxis of the ellipse of probability.

Thus, determination of the region of direction finding for a given maximum linear error is reduced to finding the region where the major semiaxis of the ellipse does not exceed the given linear error.

For calculation of semiaxes of the ellipse of probability and the angle of orientation of the major axis in the case of two radio direction finders we use formulas (11.10) and (11.11). For simplification of calculations it is conveniently to introduce these parameters (Fig. 11.7): φ — angle between median ON of line $(RDF)_1$, $(RDF)_2$, connecting the direction finders, and the line $(RDF)_1(RDF)_2$; ON — length of the median.

Let us designate $(RDF)_1(RDF)_2 = 2D$, $ON = m$. $(RDF)_1(RDF)_2$ is called the <u>goniometer base</u> of the two radio direction finders.

Area of triangle $(RDF)_1O(RDF)_2$ is determined by expression

$$\frac{D_1D_2\sin\alpha}{2} = \frac{2mD\sin\varphi}{2},$$

whence

$$\sin\alpha = \frac{2mD}{D_1D_2}\sin\varphi. \qquad (11.26)$$

From triangles $(RDF)_1ON$ and $(RDF)_2ON$ it follows that

$$\left.\begin{aligned} D_1^2 &= m^2 + D^2 + 2mD\cos\varphi, \\ D_2^2 &= m^2 + D^2 - 2mD\cos\varphi. \end{aligned}\right\} \qquad (11.27)$$

Substituting in formulas (11.10) expressions (11.26) and (11.27) and introducing designation $e = \frac{\sigma_2}{\sigma_1}$, we obtain the following equality:

$$a_0 = 2\sqrt{-\ln(1-P_0)}\,D\sigma_1 \times$$
$$\times\sqrt{\frac{\left[\left(\frac{m}{D}\right)^2+1\right]^2-4\left(\frac{m}{D}\right)^2\cos^2\varphi}{T^2+U-\sqrt{(T^2+U)^2-16\left(\frac{m}{D}\right)^2 e^2\sin^2\varphi}}}, \tag{11.28}$$

where

$$T^2 = \left[\left(\frac{m}{D}\right)^2+1\right](1+e^2),$$
$$U = 2\frac{m}{D}\cos\varphi(1-e^2).$$

From expressions (11.28) it follows that when $\varphi = 90^\circ$ the formula for the semimajor axis of the ellipse takes the form

$$a_0 = 2\sqrt{-\ln(1-P_0)}\,\sigma_1 \times$$
$$\times\frac{m^2+D^2}{\sqrt{(m^2+D^2)(1+e^2)-\sqrt{[(m^2+D^2)(1+e^2)]^2-16m^2D^2e^2}}};$$

when $\varphi = 90^\circ$ and $\sigma_1 = \sigma_2 = \sigma$,

$$a_0 = \sqrt{-\ln(1-P_0)}\;\sigma\frac{D^2+m^2}{m}, \tag{11.29}$$

when $D > m$, and

$$a_0 = \sqrt{-\ln(1-P_0)}\;\sigma\frac{D^2+m^2}{D}, \tag{11.30}$$

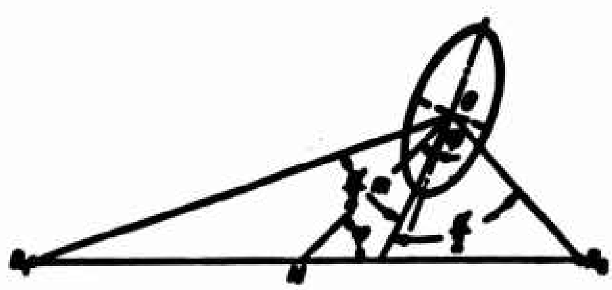

Fig. 11.7. Construction of the ellipse of probability.

when $m > D$. In all these formulas σ, σ_1, and σ_2 are expressed in radians.

When $\varphi = 90$ and $m = D$, i.e., the point of intersection of bearings is at the distance of a semibase (length of the base is equal to 2D) from both radio derection finders, then from (11.29) it follows that

$$a_0 = b_0 = 2\sqrt{-\ln(1-P_0)}\,\sigma D$$

and the ellipse of probability turns into a circle with radius $2\sqrt{-\ln(1-P_0)}\,\sigma D$. This case corresponds to minimum linear error

$$a_{0\,min} = 2\sqrt{-\ln(1-P_0)}\,\sigma D = 0{,}035\sqrt{-\ln(1-P_0)}\,\sigma^\circ D.$$

When m = 2D, i.e., for a point being an identical distance from both radio direction finders, equal to the length of the base,

$$a_0 = 5\sqrt{-\ln(1-P_0)}\,\sigma D = 2.5a_{0\,min}.$$

Let us consider methods of constructing the working zone of two radio direction finders.

Let us designate the maximum permissible linear error with probability P_{ell} given by conditions of operation by ΔL.

The condition for finding the working zone of the direction finders will be $a_0 \leq \Delta L$ for the given values of P_{ell}, σ, e, and D.

We introduce in formula (11.28) such a parameter Q that

$$a_0' = 2\sqrt{-\ln(1-P_0)}\,De\sigma Q = \Delta L. \qquad (11.31)$$

In this formula

$$Q = \frac{1}{2}\sqrt{\frac{\left[\left(\frac{m}{D}\right)^2+1\right]^2 - 4\left(\frac{m}{D}\right)^2\cos^2\varphi}{\left(\frac{m}{D}\right)^2+1-\sqrt{\left[\left(\frac{m}{D}\right)^2+1\right]^2-4\left(\frac{m}{D}\right)^2\sin^2\varphi}}}, \qquad (11.32)$$

when $\sigma_1 = \sigma_2 = \sigma$;

$$Q = \sqrt{\frac{\left[\left(\frac{m}{D}\right)^2+1\right]^2 - 4\left(\frac{m}{D}\right)^2\cos^2\varphi}{T^2+U-\sqrt{(T^2+U)^2-16\left(\frac{m}{D}\right)^2 e^2\sin^2\varphi}}}, \qquad (11.33)$$

when $\sigma_1 = \sigma$, $\sigma_2 = e\sigma$.

For given ΔL and P_{ell}, and also σ, e, and D, for the considered radio direction finders it is possible to calculate by (11.31) Q, which depends on φ, e, and $\frac{m}{D}$. Knowing Q and e, it is possible, using (11.32) or (11.33), to calculate the dependence of $\frac{m}{D}$ on φ and to obtain initial data for construction of working zones of the two radio direction finders.

For construction of boundaries of the working zone of direction finders one should:

— to line $(RDF)_1(RDF)_2$, connecting the radio direction finders, pass medians at various angles φ;

— on medians CA plot segments m.

Obtained final points of medians determine boundaries of the working zone of direction finding.

If $e = 1$, i.e., accuracies of both radio direction finders are identical, then boundaries of the zone are symmetric with respect to the perpendicular to center line $(RDF)_1(RDF)_2$; therefore, for angles φ and $180^\circ - \varphi$ we obtain one and the value of $\frac{m}{D}$. If $e \neq 1$, i.e., accuracies of radio direction finders differ, the curve of the working zone is asymmetric. For every value of φ there are two values: $\frac{m}{D}$ and m, bounding on two sides the working zone of direction finding.

To facilitate calculations we give Tables 11.3 and 11.4 of values of Q for various $\frac{m}{2D}$ and φ when $e = 1$ and $e = 2$, calculated by V. V. Shirkov [11.2].

Table 11.3. Value of Parameter Q for Case of Equally-Exact Work of Direction Finders ($e = 1$)

$\frac{m}{2D}$	φ°					
	15 or 165	30 or 150	45 or 135	60 or 120	75 or 105	90
0.04	13.6	7.08	5.01	4.00	3.66	3.54
0.1	6.00	3.51	2.53	2.10	1.97	1.84
0.2	3.15	1.74	1.32	1.14	1.06	1.03
0.3	1.88	1.17	0.96	0.86	0.82	0.80
0.4	1.10	0.96	0.88	0.82	0.76	0.72
0.5	0.90	0.97	0.92	0.87	0.79	0.71
0.6	1.34	1.14	1.06	0.90	0.81	0.86
0.7	2.00	1.43	1.26	1.16	1.08	1.06
0.8	2.82	1.81	1.52	1.38	1.29	1.26
1.0	4.83	2.70	2.30	1.93	1.81	1.77
1.2	7.22	4.04	3.06	2.64	2.45	2.39
1.4	10.10	5.47	4.09	3.48	3.21	3.13
1.6	13.34	7.16	5.77	4.46	4.00	3.98

During construction of the zone for unequally exact radio direction finders one should consider that $(RDF)_1$ is the more exact radio direction finder and that angles φ should be plotted from $C(RDF)_1$.

With equally exact radio direction finders the maximum range of direction finding, or least error for any distance from the middle of the line of the goniometer base, is obtained along the perpendicular to the middle of the line of the goniometer base.

It is possible to construct boundaries of the region of the working zone, proceeding from obtaining on the boundary of region of the given mean quadratic linear error.

Table 11.4. Value of Parameter Q for Case $e = 2$

$\frac{m}{2D}$	φ°										
	15	30	45	60	75	90	105	120	135	150	165
0.05	22.80	11.80	8.30	6.70	5.80	5.60	5.70	6.26	7.60	10.6	20.5
0.1	11.70	6.13	4.33	3.82	3.11	2.93	2.91	3.12	3.66	[illegible]	9.32
0.2	6.18	3.26	2.42	2.03	1.81	1.67	1.50	1.60	1.74	2.18	3.80
0.3	3.72	2.32	1.80	1.67	1.51	1.37	1.24	1.16	1.16	1.32	2.04
0.4	2.23	1.93	1.76	1.62	1.46	1.30	1.17	1.00	0.94	[illegible]	1.21
0.5	1.86	1.93	1.86	1.73	1.50	1.41	1.22	1.00	0.92	0.97	0.90
0.6	2.67	2.26	2.00	1.96	1.76	1.64	1.39	1.19	1.10	1.16	1.36
0.7	3.97	2.76	2.47	2.26	2.06	1.85	1.64	1.47	1.40	1.52	2.09
0.8	5.60	3.54	2.96	2.64	2.38	2.16	1.96	1.80	1.79	2.00	3.05
1.0	9.15	5.34	4.16	3.58	3.18	2.93	2.73	2.64	2.75	3.29	5.53
1.2	13.46	7.82	5.66	4.75	4.26	3.91	3.72	3.54	3.90	5.00	8.72
1.4	18.72	10.17	7.27	6.07	5.46	5.00	4.80	4.90	5.50	7.06	12.60
1.6	24.33	12.96	9.40	7.76	6.86	6.40	6.23	6.43	7.26	9.46	17.29

For mean quadratic linear error of two radio direction finders we obtained

$$R_0=\frac{\sqrt{D_1^2\sigma_1^2+D_2^2\sigma_2^2}}{\sin\alpha_{12}}\ (\sigma_1 \text{ and } \sigma_2 \text{ in radians}) =$$
$$=\frac{0.0175}{\sin\alpha_{12}}\sqrt{D_1^2\sigma_1^2+D_2^2\sigma_2^2}\ (\sigma_1 \text{ and } \sigma_2 \text{ in degrees}).$$

If we express R_0 depending upon m and φ (see Fig. 11.7), for $\sigma_1 = \sigma_2 = \sigma$ the formula for R_0 will take form [11.3]

$$R_0=\frac{\sqrt{2}\sigma D}{\sin\varphi}\sqrt{\left[\left(\frac{m}{2D}+\frac{D}{2m}\right)^2-\cos^2\varphi\right]\left[\left(\frac{m}{D}\right)^2+1\right]}.$$

In order to construct the boundary of the zone of direction finding, for which mean quadratic linear error R_0 will not exceed a given value ΔL, i.e., $R_0 \leq \Delta L$, we use the series of curves of Fig. 11.8, on which there are depicted dependences $\frac{R_0}{2\sigma D} = f(\frac{m}{2D})$ for various values of φ. Minimum error ΔL_{min} is obtained at point $\varphi = 90^\circ$, $m = 0.7D$, where $\alpha_{12} = 109^\circ$, $\Delta L_{min} = 0.032\sigma D$, σ in degrees.

Frequently for the line of the working zone of direction finding we take two circles with radius 2D, passing through direction finders $(RDF)_1$ and $(RDF)_2$. At all points of these circles bearings cross at angles 30° (external circle) and 150° (internal circle).

Calculation shows that linear error for boundaries of the first circumference in its central part is approximately equal to $7a_{0\ min}$; linear error for boundaries of the second circle is equal to $2a_{0\ min}$.

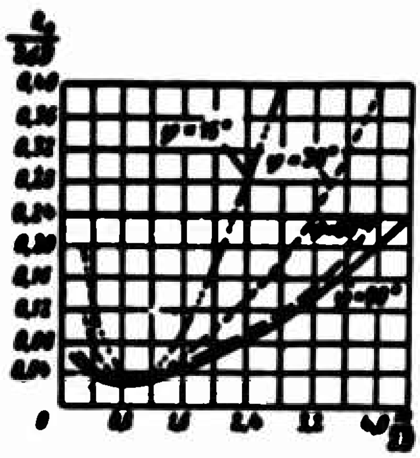

Fig. 11.8. Dependences of $\frac{R_0}{2\sigma D}$ on $\frac{m}{D}$ for various values of φ.

In Table 11.5 there are calculated linear errors for the central part of circles of various angles of intersection of bearings.

In Fig. 11.9 there are constructed for two radio direction finders contours of zones of direction finding on the boundary of which there is observed constant maximum error (a_0 – semimajor axis of the ellipse of probability) or constant mean quadratic linear error (R_0 – circular error). On the same figure there are drawn neighborhoods of constant angles of intersection of bearings (α_{12} = const).

In the described calculations it was assumed that mean quadratic errors of the radio direction finder kept constant within limits of the whole zone of direction

Table 11.5. Linear Errors $\frac{a_0}{a_{0\,min}}$ for Various Angles of Intersection of Bearings

Angle of intersection of bearings, degrees	90	60 and 150	45 and 160	10 and 170	28.5
a_0/a_0 min	1	2	3.4	7.8	13

finding. In fact these errors vary with distance, and when we allow for this construction is considerably complicated.

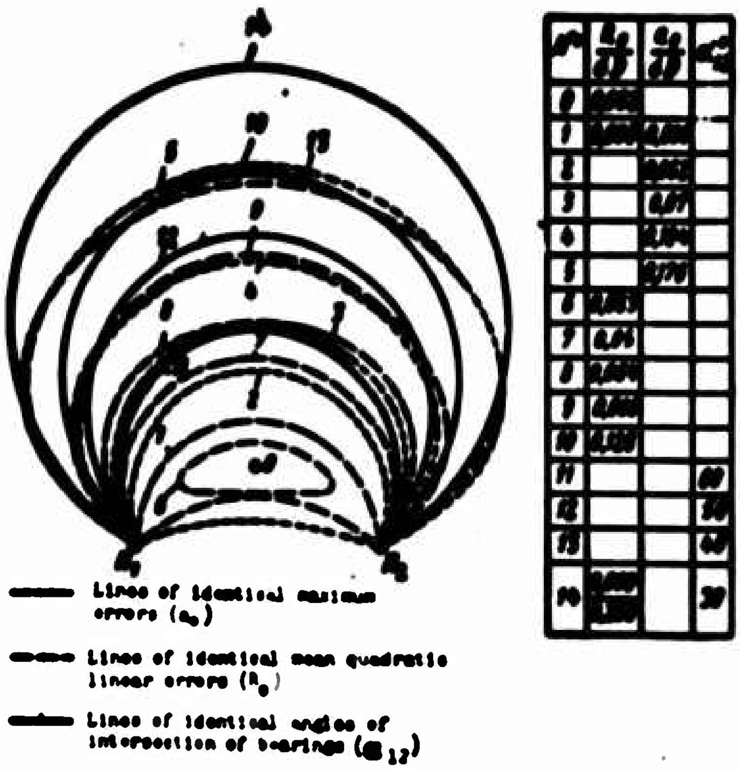

Fig. 11.9. Zones of radio direction finding for two radio direction finders.

If direction finding is carried out on medium and long waves, the mean quadratic errors are kept approximately constant for the whole working zone. There are limiting distances, beyond which direction finding becomes unreliable. Therefore, after construction of the working zone by mean values of mean quadratic error one should sketch around each radio direction finder maximum regions of direction finding, beyond which direction finding becomes unsatisfactory, and thus cut off areas which are beyond the boundaries of these regions.

With direction finding of short waves is possible to speak of varying angular mean quadratic error at various distances from the radio direction finder. Most simple will be the following characteristic of direction finding:

— at a certain radius r direction finding is impossible (zone of silence);

— at distances from r to R_1 radio direction finders work with reduced accuracy characterized by σ' (zone of steeply incident waves);

— at distances from R_1 to R_2 radio direction finders work with normal accuracy, characterized by σ''.

With such assumptions for determination of the region covered by two radio direction finders it is necessary to construct zones of direction finding for the following conditions:

1) both radio direction finder work with identical accuracy σ';

2) both radio direction finder work with identical accuracy σ'';

3) first direction finder has accuracy, characterized by σ'; and the second, characterized by σ'';

4) first radio direction finder has accuracy σ''; and the second, accuracy σ'.

Furthermore, it is necessary to construct circles with radius r and R_2, separating zones, within which direction finding is impossible.

As a result of such construction we obtain a region of complex outline, sometimes embracing several sections, not interconnected. Individual curves of arcs, bounding the region, intersect at acute or obtuse angles, i.e., there is not a smooth transition of one curve into the other. This is caused by the fact that there is allowed intermittent change of conditions of direction finding. In reality conditions of direction finding vary smoothly; this permits us after construction to somewhat round sharp transitions of one curve, bounding the working zone of direction finding, into another.

When situating a group of radio direction finders for servicing a certain region it is necessary to take into account requirements of obtaining permissible linear error and normal passage of radio waves from the regions to radio direction finders. For more on situating of a group of radio direction finders see [11.1].